ECE445
SENIOR DESIGN LABORATORY

DESIGN DOCUMENT

Low-Latency Analog Differential Equation Solver

Team #42

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1 Introduction

1.1 Problem Statement

Ordinary Differential Equations (ODEs) are fundamental tools for modeling dynamic systems in engineering, including mechanical vibration systems, electrical circuits, and control systems. These systems evolve continuously over time and are commonly described by equations involving derivatives of system variables.

Traditionally, ODEs are solved using digital numerical methods such as Euler or Runge–Kutta algorithms. While these methods are flexible and widely used, they require iterative computation, which produces computational latency and limits real-time performance.

In applications such as real-time system simulation, rapid prototyping, and control system visualization, low-latency solutions are highly desirable. Therefore, an alternative approach is to directly implement differential equations using analog hardware.

This project addresses this need by developing an analog differential equation solver that computes system responses in continuous time using operational amplifier (op-amp) circuits. The goal is to eliminate computational delay and enable real-time observation of system dynamics.[2]

1.2 Solution Overview

The proposed solution is an analog computing system that implements differential equations using electrical circuits. In this approach, system variables are represented as voltage signals, and mathematical operations are mapped to circuit components.

- Differentiation is implemented using op-amp differentiator circuits
- Addition and scaling are implemented using summing amplifiers and resistor networks
- Feedback loops enforce the structure of the differential equation

The entire system forms a closed-loop analog computation network. When an input signal is applied, the circuit continuously produces an output voltage corresponding to the solution of the differential equation.

1.3 High-Level Requirements List

To ensure successful implementation of the analog differential equation solver, the following high-level requirements are defined:

Functional Requirements

1. The system should implement a first- or second-order ordinary differential equation using analog circuits.
2. The system should correctly compute time derivatives using a differentiator circuit.
3. The system should perform weighted summation of signals to represent equation coefficients.

4. The system should accept external input signals such as step, sinusoidal, and square wave inputs.

5. The system should generate a continuous-time output signal representing the solution of the differential equation.

Performance Requirements

1. The system should operate in real time with negligible computational latency.

2. The circuit should remain stable within the intended frequency range.

3. The output waveform should match theoretical system behavior within acceptable tolerance.

Hardware Requirements

1. The system should use operational amplifiers as the primary computing elements.

2. The circuit should be powered by a dual supply.

3. The system should interface with a function generator for input and an oscilloscope for output observation.

2 Design

2.1 Block Diagram

The system operates as a closed-loop analog computer. The input signal $u(t)$ is first scaled and combined with feedback signals from the system states. The resulting signal represents the second derivative $\ddot{x}$, which is then integrated twice to produce $\dot{x}$ and x. These state variables are fed back into the summation circuit, forming a continuous-time feedback loop that solves the differential equation in real time.

The system consists of several subsystems, each responsible for implementing a specific function required for solving the differential equation.

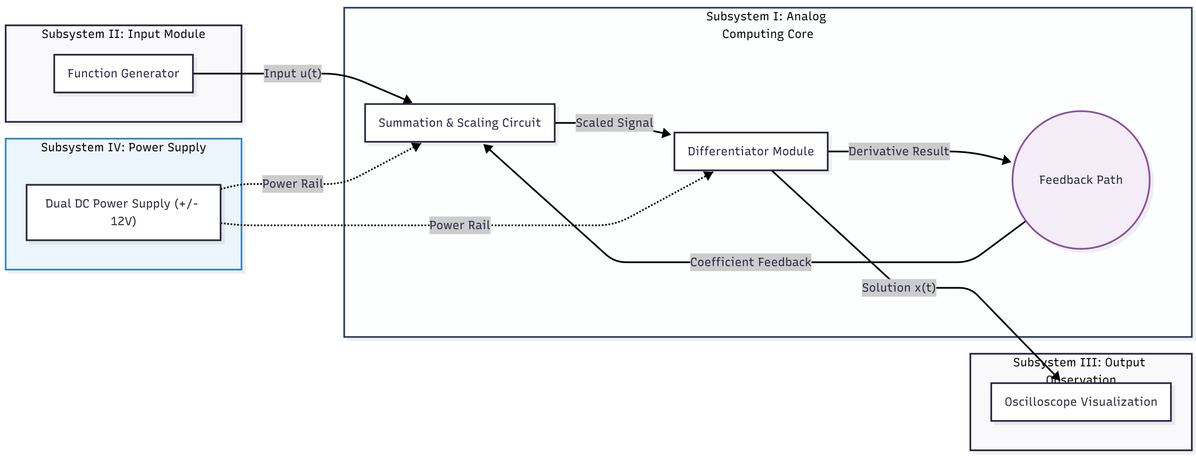

Figure 1: The block diagram of the Analog Differential Equation Solver

2.2 Analog Computing Core

The Differentiation System is the core of the solver, responsible for generating derivative terms and combining them according to the target differential equation. The system is implemented using analog circuits that directly map mathematical operations into physical components.

2.2.1 Hardware Support

Operational Amplifier (TL084) The TL084 quad operational amplifier is used as the core analog computing component. It contains four independent op-amps, which are configured to implement integrators and a summing amplifier.

Two amplifiers are used to form cascaded integrators that generate the system states (i.e., velocity and displacement), while another is used to combine weighted signals corresponding to the differential equation. This configuration allows the circuit to directly represent the mathematical model.

The TL084 operates with a dual power supply (e.g., ±12V), enabling both positive and negative voltage signals. Its analog nature allows continuous-time computation with low latency, making it suitable for real-time dynamic system implementation.

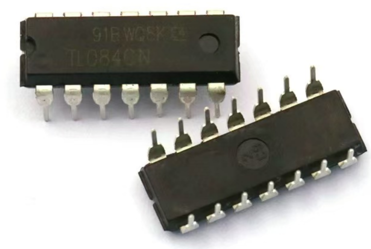

Figure 2: Quad Op-Amp (TL084) used in the differentiation system

Resistor and Capacitor Application Resistors and capacitors are essential for defining the behavior of the analog computing circuit. In summing amplifiers, resistor ratios determine the coefficients of the differential equation, allowing weighted combinations of signals to be implemented.

For a second-order system:

$$\ddot{x} + a\dot{x} + bx = cu(t) \tag{1}$$

it can be rewritten as:

$$\ddot{x} = -a\dot{x} - bx + cu(t) \tag{2}$$

This form is realized using an inverting summing amplifier, whose output is:

$$V_{out} = -\left(\frac{R_f}{R_1}x' + \frac{R_f}{R_2}x + \frac{R_f}{R_3}u(t)\right) \tag{3}$$

By comparing the two expressions, the system coefficients are directly determined by resistor ratios:

$$a = \frac{R_f}{R_1}, \quad b = \frac{R_f}{R_2}, \quad c = \frac{R_f}{R_3} \tag{4}$$

In integrator circuits, capacitors work together with resistors to perform mathematical integration. The output of the integrator is given by:

$$V_{out} = -\frac{1}{RC}\int V_{in}\,dt \tag{5}$$

Thus, the resistor-capacitor network defines the time constant $\tau = RC$, which determines the dynamic response speed of the system.

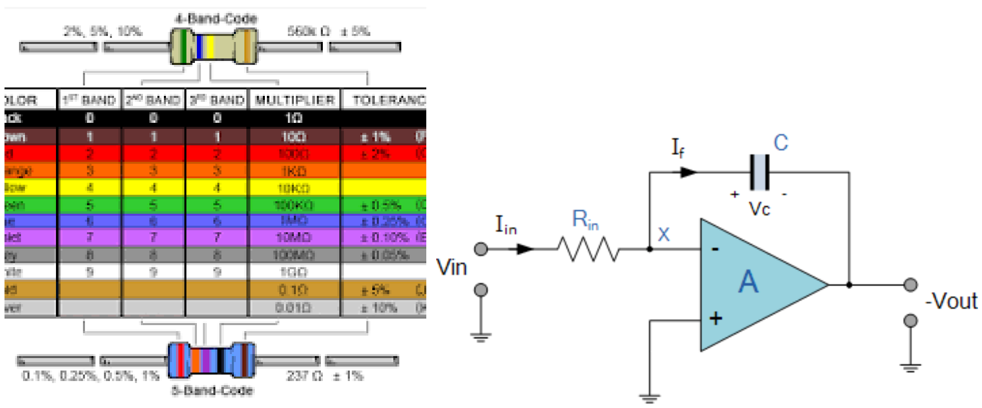

Figure 3: Resistors and their role in op-amp integrator and summing circuits

2.2.2 Mathematical Model

A typical second-order system such as a mass-spring-damper system is described by:

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{6}$$

The analog circuit implements this model by forming a feedback loop that satisfies:

$$\ddot{x} = -a\dot{x} - bx + cu(t) \tag{7}$$

where the coefficients are physically realized using resistor ratios, and integration is achieved through RC-based op-amp integrators. This enables the circuit to continuously compute the system response in real time.

2.2.3 Model-to-Circuit Mapping

The relationship between the Simulink model, the analog circuit implementation, and the governing differential equation can be directly established.

The Simulink model represents the system using two cascaded integrator blocks and a summation block. The summation block combines the feedback signals and the external input to produce the second derivative of the state variable. The integrators then successively generate the first derivative and the state variable itself.

In the analog circuit, this relationship is implemented using operational amplifiers. The summation block is realized by an inverting summing amplifier, which combines the signals x, $\dot{x}$, and $u(t)$ with appropriate weights. The output of this block represents $\ddot{x}$.

The integrator blocks are implemented using op-amp integrator circuits. The first integrator converts $\ddot{x}$ into $\dot{x}$, and the second integrator converts $\dot{x}$ into x. These outputs are then fed back into the summing amplifier, forming a closed-loop system.

The coefficients of the differential equation are mapped to resistor ratios in the summing amplifier:

$$a = \frac{R_f}{R_1}, \quad b = \frac{R_f}{R_2}, \quad c = \frac{R_f}{R_3} \tag{8}$$

Thus, the Simulink model, the analog circuit, and the mathematical formulation are fully consistent. The signal flow in the Simulink diagram directly corresponds to the physical signal paths in the circuit, enabling a one-to-one mapping between simulation and hardware implementation. This demonstrates that the circuit functions as a physical solver of the differential equation.

2.3 Input Signal System

2.3.1 Hardware Support

This system uses a function generator to provide the external excitation $u(t)$. It can produce step signals, sinusoidal waves, or square waves to simulate various external forces $F(t)$ applied to the modeled dynamic system. The amplitude and frequency of the input signal are adjustable, allowing the system to be tested under different excitation conditions such as step response, sinusoidal steady-state response, and transient dynamics.

2.4 Output Observation System

2.4.1 Hardware Support

The solution of the differential equation is represented as a time-varying voltage signal at the output of the circuit. This voltage corresponds to the system state (e.g., displacement $x(t)$) and can be directly measured.

An oscilloscope is used to observe the output signal in real time. By connecting the output node of the circuit to the oscilloscope, the dynamic behavior of the system can be visualized as a waveform.

Typical observed behaviors include oscillations, exponential decay, and steady-state responses, depending on the system parameters and input signal. For example, under sinusoidal excitation, the output may exhibit forced oscillations, while in the absence of input, the system may show natural response characteristics.

The oscilloscope allows direct comparison between theoretical predictions and experimental results. Key features such as amplitude, frequency, phase, and transient response can be analyzed from the measured waveform.

In addition, multiple channels of the oscilloscope can be used to simultaneously observe different signals, such as $x(t)$ and $\dot{x}(t)$, providing deeper insight into the system dynamics.

2.4.2 Output Characteristics

The output signal reflects the real-time solution of the implemented differential equation. Since the computation is performed in continuous time, the waveform is smooth and free from discretization effects.

The system response varies with different input signals and parameter settings. By adjusting the circuit parameters (e.g., resistor values), different dynamic behaviors such as underdamped, overdamped, or critically damped responses can be observed.

This real-time visualization provides an intuitive understanding of the system behavior and demonstrates the effectiveness of analog computation in solving differential equations.

2.5 Power Supply System

2.5.1 Hardware Support

To allow the operational amplifiers to process signals that swing both positive and negative around a zero-volt ground, the system is powered by a dual DC power supply providing ±12V.

2.6 Tolerance Analysis

The accuracy of this analog solver is primarily limited by the tolerance of passive components. For a differentiator with time constant $\tau = RC$, the relative error is:

$$\frac{\Delta \tau}{\tau} = \sqrt{\left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta C}{C}\right)^2} \tag{9}$$

Using 1% resistors and 5% capacitors, the expected error is approximately 5.1%. To achieve the high-level requirement of consistency with theoretical behavior, high-precision metal film resistors and film capacitors will be used, complemented by multi-turn potentiometers for calibration.

2.7 Requirements and Verification

Requirements	Verification
1. **Accurate Differentiation**: The circuit must correctly compute the time derivative within the target frequency range (10Hz – 1kHz).	1. Input a $1V_{pp}$ triangle wave. Use an oscilloscope to verify the output is a square wave with calculated amplitude $V_o = -RC(dV/dt)$.
2. **Correct Implementation**: Summing circuits must scale signals according to coefficients with < 5% error.	2. Apply DC test voltages to each input and measure the output using a digital multimeter to confirm the gain matches R_f/R_{in}.
3. **Real-Time Response**: The system must show no observable phase lag or delay compared to the theoretical model.	3. Compare the input excitation and output response simultaneously on a dual-channel oscilloscope to ensure zero computational latency.
4. **Theoretical Consistency**: Output waveforms must match predicted damping and oscillation patterns.	4. Plot the theoretical solution in Python/MATLAB and overlay the captured oscilloscope data to verify overlap within a 10% tolerance band.

3 Cost

Description	Quantity	Unit Cost	Total
TL084 Quad Op-Amp	5	$1.50	$7.50
Precision Resistor Set (1%)	1	$15.00	$15.00
Precision Capacitor Set (5%)	1	$20.00	$20.00
Multi-turn Potentiometers	10	$2.00	$20.00
Custom PCB Manufacturing	2	$25.00	$50.00
Breadboards & Connectors	1	$30.00	$30.00
Total Parts Cost			**$142.50**

4 Schedule

Week	Yishan Sheng	Jiachang Wang	Yanzi Li	Tianyue Jia
3/16	Finalize ODE model	Research op-amp specs	Initial circuit design	Component selection
3/23	LTspice simulations	Component procurement	Breadboard prototyping	Power supply testing
3/30	Subsystem integration	PCB Layout design	Verification of gain	Differentiator tuning
4/6	PCB soldering	Signal integrity test	Accuracy verification	Data collection
4/13	Tolerance analysis	Final system debug	Compare with MATLAB	Finalizing plots
4/20	Final Report writing	Video demonstration	Documentation	Presentation prep

5 Ethics and Safety

5.1 Ethics

Our project aligns with the IEEE Code of Ethics[1], specifically the commitment to uphold the highest standards of integrity and responsible conduct. As we are building a tool for mathematical modeling and simulation, we ensure that:

- **Honesty in Reporting:** All experimental data, including the error margins and tolerance analysis, are reported honestly without manipulation to match theoretical ideals.
- **Technical Competence:** We acknowledge the limitations of analog computing, such as noise floor and component drift, and provide clear documentation on the system's operational range.
- **Conflict of Interest:** No conflicts of interest exist in the development of this project, and all sources of components and prior research are properly acknowledged.

5.2 Safety

Safety is a primary concern given the use of electrical equipment and power supplies.

- **Electrical Safety:** The system operates on a ± 12V DC dual power supply. While these voltages are generally safe for human contact, short circuits can lead to component failure or minor burns. All power rails will be clearly labeled, and current-limiting features of the bench power supply will be utilized during testing.
- **Equipment Safety:** The use of oscilloscopes and function generators follows standard laboratory protocols. Proper grounding will be maintained to prevent equipment damage and ensure accurate signal measurement.
- **Soldering Safety:** During PCB assembly, lead-free solder will be used where possible. Soldering will be conducted in a well-ventilated area using fume extractors to prevent the inhalation of hazardous vapors.
- **Capacitor Handling:** High-value capacitors used in the integrator/differentiator stages will be discharged before circuit modification to prevent accidental energy release.

References

[1] IEEE Global Initiative on Ethics of Autonomous Systems. Ethically aligned design: A vision for prioritizing human well-being. *Technical report, IEEE Standards Association*, 2021.

[2] ElAli, T., et al. "An Analog Computer To Solve Any Second Order Linear Differential Equation With Arbitrary Coefficients." *Innovative Algorithms and Techniques in Automation, Industrial Electronics and Telecommunications*. Dordrecht: Springer Netherlands, 2007. 449-451.