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浙江大学伊利诺伊大学厄巴纳香槟校区联合学院

ECE 445: Senior Design

Project Proposal

Analog Computer ODE Solver

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1 Introduction

1.1 Background and Motivation

In the field of scientific and engineering computations, the demand for efficient and versatile computing solutions has always been high. Traditional digital computers have dominated computing for decades. However, analog computers offer unique advantages in certain applications.

Analog computers operate based on continuous physical quantities, such as voltage or current, which can represent mathematical variables directly[11]. This property allows them to perform computations in a more ”natural” way for some problems, especially those involving continuous-time systems.

Ordinary Differential Equations (ODEs) [3] are prevalent in many scientific and engineering disciplines. For example, in physics, ODEs are used to describe the motion of objects [8], the behavior of electrical circuits, and the diffusion of substances. In engineering, they are essential for designing control systems, analyzing mechanical vibrations, and predicting chemical reactions. Solving ODEs accurately and quickly is crucial for these applications.

The motivation behind creating an analog computer with ODE solvers lies in exploring an alternative computing paradigm that can potentially offer faster solutions for ODE-related problems, especially in real-time applications where digital computers may face limitations due to their discrete-time nature and the need for numerical approximations. Additionally, by adding features like the ability to change the circuit structure using switches, we aim to expand the functionality of the analog computer beyond just ODE solving. This could enable it to handle other types of equations and perform operations such as Taylor series expansion, providing a more versatile and powerful tool for scientific research and education.

1.2 Literature Review

Previous research on analog computers has shown their effectiveness in solving differential equations. Early analog computers, such as the differential analyzer [16] developed in the 1930s, were large mechanical devices that used wheels and disks to perform integrations and solve ODEs. With the advent of electronics, analog computers became more compact and precise.

Studies have demonstrated that analog circuits can be designed to solve first-order [4] and higher-order ODEs [5, 6]. For example, a simple RC (resistor-capacitor) circuit can be used to approximate the solution of a first-order linear ODE. More complex circuits, involving operational amplifiers, can handle non-linear ODEs.

Regarding the use of switches in analog circuits, some research has explored their application in reconfigurable analog circuits. These circuits can change their functionality based on the state of the switches. However, the design of switches in an analog computer for the purpose of changing the circuit to solve different types of equations or perform different operations like Taylor series expansion is still an area with room for exploration. There are few existing works that specifically address the integration of switch-based reconfiguration for such a wide range of computational tasks in an analog computer.

1.3 Challenges and Objectives

1.3.1 Potential Challenges

- **Switch Design**: Designing switches that can accurately and reliably change the circuit structure is a major challenge. The switches need to have low resistance when closed to avoid affecting the analog signals in the circuit and high resistance when open to ensure that the unwanted parts of the circuit are effectively isolated.

- **Accuracy and Precision**: Analog computations are prone to errors due to component tolerances, such as the tolerance of resistors and capacitors [1]. Ensuring that the analog computer can provide accurate solutions for a wide range of equations, especially non-linear ones, is difficult.

- **Scalability**: As the complexity of the equations and operations to be performed increases, scaling up the analog computer to handle more complex tasks without sacrificing performance becomes a challenge.

1.3.2 Project Objectives

- **ODE Solving**: Develop an analog computer capable of accurately solving a variety of ODEs, including linear and non-linear ODEs.
- **Function Expansion**: Enable the analog computer to perform other functions, such as solving general algebraic equations and calculating Taylor series expansion through switch-based circuit reconfiguration.
- **Optimization**: Optimize the design of the analog computer in terms of accuracy, speed, and component cost.

1.4 High-level Requirements List

- Analog computer circuit must be hardware-concept programmable.
- The whole system should not contain FPGA or other high-level programmable devices.

2 Design and Requirements

In order to design an analog computer for solving mathematical problems like ODEs, we plan to use operation amplifiers to build integrators, adders, and multipliers to create a feedback loop [8]. We will convert the ODE into specific electrical signals, such as voltage, input these signals into the solving circuit, and then convert the output back to obtain the solution of the ODE, which will be shown directly to the user via an oscilloscope. From there, we build a useful mathematical problem solver. Some idea of our design refers to the paper *DPAC: Digitally Programmable Analog Computer* [15].

In order to implement several functionalities, we plan to make the analog computer programmable. We propose two steps to implement it. Firstly, we will design a programmable microcontroller to receive the input equations and then control switches in circuits to change it to be more adaptable to solve the particular type of equation. Secondly, for particular equation types, we propose using variable resistors to deal with different coefficients. Digital potentiometers and multipliers may serve as variable resistors.

We divide the whole system into four subsystems. A power supply is responsible for providing stable positive and negative levels, zero level, and stable voltage with a certain ratio to the controller and analog circuits. A programmable microcontroller is used to receive inputs from the user side and control the change of analog circuits. An analog computer circuit is a feedback circuit using block modules connected in a specific way; the feedback circuit simulates the equations to be solved (e.g., ODEs) through voltage values and is the core part of the analog computer. An I/O system needs to receive the equations from the user, convert the digital signals into analog signals and transmit them to the circuit, and then output the output of the circuit in real time using an oscilloscope or convert it into digital signals.

2.1 Block Diagram

In this section, we will explain how our proposed analog computer will work and what subsystem it will consist of. The following Figure 1 is the holistic system of the analog computer.

2.2 Required Subsystems

This section is a short description of the major subsystems that would be required in this project. Implementation details of the subsystems will be explained in the following sections.

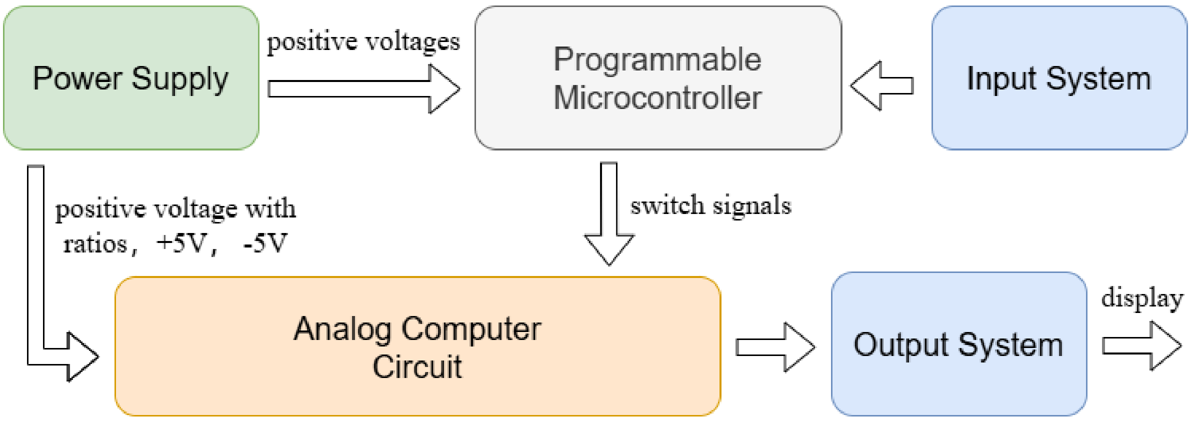

Figure 1: Block diagram for our proposed analog computer

2.2.1 Power Supply

In our design, the microcontroller requires access to a fixed positive electrode. We used analog switches and operational amplifiers that have $+5V$ and $-5V$ bipolar supplies. The multiplier we use also requires a specific ratio of voltage. Therefore, we need a circuit to provide the positive voltages, $+5V$ and $-5V$, and adjustable voltages of specific ratios needed by the microcontroller and circuits.

2.2.2 Programmable Microcontroller

The Analog computer circuit contains a large number of analog switches to control the form of the differential equations to be solved and to control the coefficients of the differential equations to be solved using a large number of variable resistors or by changing the ratio of the multiplier access voltages. These switches can be configured using a microcontroller that uses an Arduino to receive inputs from the user side and communicate with the switches and set their state to on or off, thus ensuring the programmability of the overall system.

2.2.3 Analog Computer Circuit

The analog computer is divided into three main stages: **1) the coefficient amplification stage**, **2) the adder stage**, and **3) the integrator stage**. The coefficient amplification stage amplifies each signal based on the coefficients in the equation to be

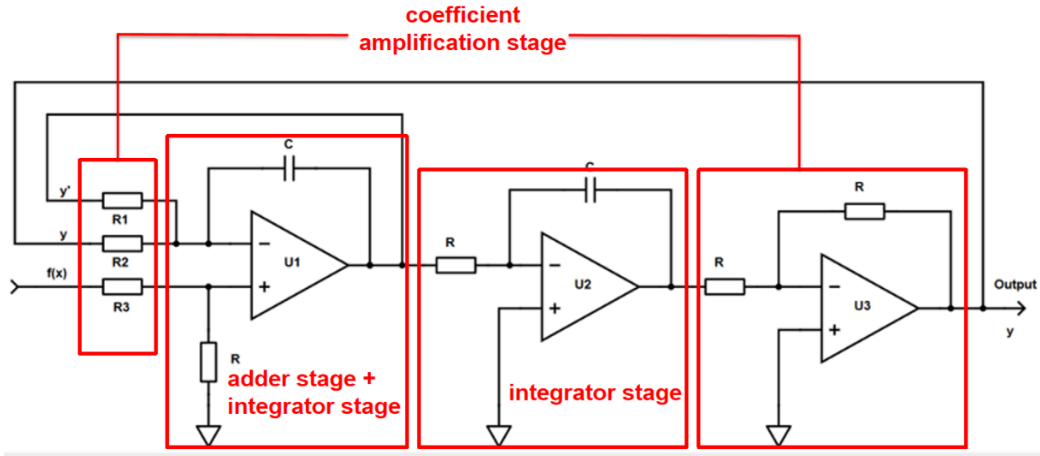

Figure 2: An example of circuit implementation solves the certain type of ODEs: $y'' = ay' + by - cu$

solved by using amplifiers, variable resistors and multipliers. The integrator stage uses an integrator to solve for the differential signals needed in the ODEs. The adder stage combines the amplified signals linearly by adding the amplified signals to $u(t)$ based on the form of the equation. Finally, these signals are integrated into the feedback circuit according to the form of the equation to be solved, which can be converged very quickly by the initial voltage to the solution of the equation.

2.2.4 I/O System

For the input system, we propose to use the interactive interface of Arduino to realize the interaction with the user. After the equations to be solved are input as digital signals, the form and coefficients of the equations are automatically determined and extracted and transmitted to the microcontroller. For the output system, the solution of the circuit can be directly displayed as an analog signal by an oscilloscope in real time, or it can be converted into a numerical signal again and returned to the computer for recording.

2.3 Modules Design

All computational primitives used in the circuit (e.g., adders, multipliers, integrators) are constructed by us from operational amplifiers. The design and details are described below.

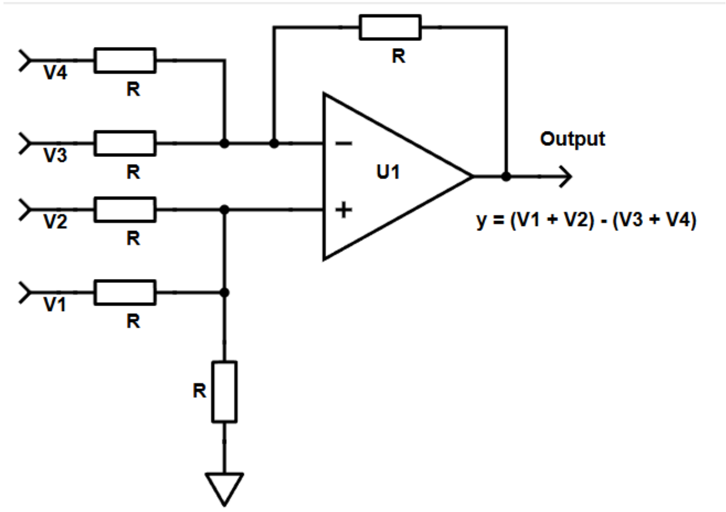

Figure 3: Op-amp Adder-Subtractor Component Circuit

2.3.1 Op-amp Adder-Subtractor

With an op-amp configured in differential mode, signals can be applied simultaneously to both the inverting and non-inverting inputs of the op-amp to perform addition and subtraction operations. Extra adding or subtracting terms can be added by simply adding more input resistors to the inputs of the op-amp.

The above Figure 3 will represent an equation of:

$$y = (V1 + V2) - (V3 + V4) \tag{1}$$

2.3.2 Op-amp Multiplier-Divider

Multiplication and division using op-amps are more involved. One technique to achieve this is to exploit the exponential nature of semiconductors that occurs around their non-linear region "knee point" for specifically producing logarithmic and anti-logarithmic converters. Transistors are chosen here in favour of diodes as they typically exhibit a wider decade current logging range.

We know that the log of a product is the sum of the log factors and that the log of a quotient is the difference of the log factors. By taking the log of each input signal, combining them, and then finally taking the anti-log, an adequate multiplier-divider can be realized. Op-amps U1, U2, and U3 are configured as log amplifiers, while U4 is

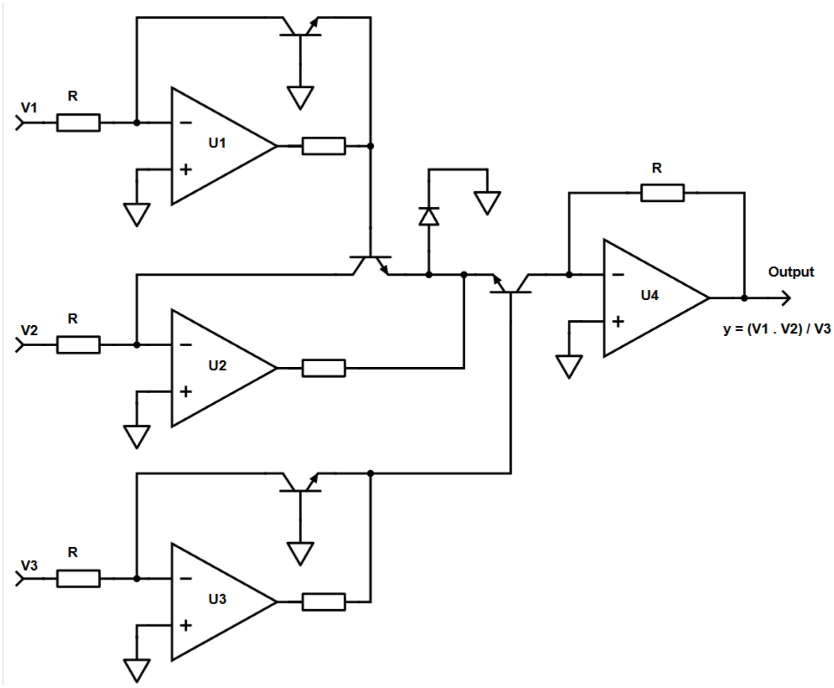

Figure 4: Op-amp Multiplier-Divider Component Circuit

configured as an antilog amplifier. It is worth remembering the limitations of this circuit as a single quadrant device only, and thus, it only functions correctly with positive input voltages.

The circuit in Figure 4 in the figure will represent an equation of:

$$y = antilog[\log(V1) + \log(V2) - \log(V3)] = \frac{V1 \cdot V2}{V3} \tag{2}$$

2.3.3 Op-amp Integrator

The circuit acts as a basic integrator, since all current sourced from V arriving at the inverting input of the op-amp must exit through the feedback path, and the voltage across the capacitor is proportional to the integral with respect to time of the capacitor charging current.

The Figure 5 will perform as the following equation:

$$y = -\frac{1}{CR} \int V dt \tag{3}$$

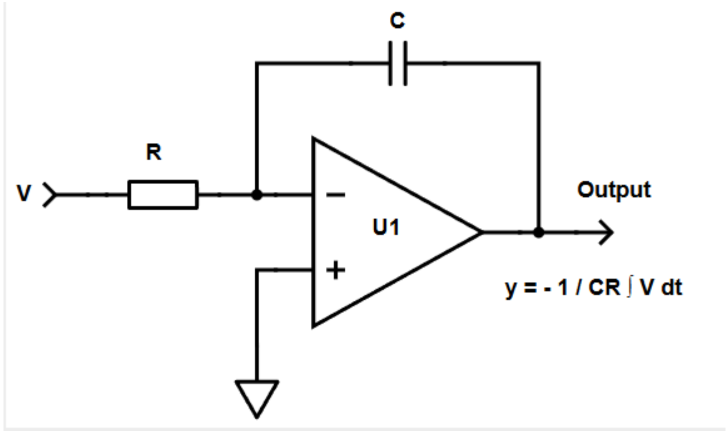

Figure 5: Op-amp Integrator Component Circuit

2.4 Implement Programmability

Arduino is an open-source electronic prototyping platform that has a key role in this project. It is programmed in a simple C/C++ language with an intuitive Integrated Development Environment (IDE). In this analog computer project, Arduino serves as a programmable microcontroller that receives equation information from the user and accurately controls a large number of analog switches in the circuit, thus adjusting the circuit to solve different differential equations and ensuring that the system is highly programmable.

In addition, in order to solve the equations with different coefficients, we propose two solutions: using a variable resistor and using a multiplier. Digital potentiometers are commonly used in PCBs as electronic components that can control the value of resistors through digital signals, which can realize the precise adjustment of circuit gain, voltage division, and other parameters. But the process of changing the resistance value with digital signals is one that may bring excessive time delay. We will also try to use photosensitive resistors and piezoresistors. Using the multiplier mentioned in the previous section to amplify the signal directly based on the coefficients can change the circuit more quickly, but it introduces too many level inputs and creates a more complex circuit.

For the switches of the control circuit, we chose the CD4051 chip, which is a single-ended 8-channel multiswitch commonly used in PCBs to ensure high-speed operation.

2.5 Simulation and PCB Building

After completing the circuit design, we will use the Simulink tool in Matlab to simulate the analog computer. Starting from solving first-order ordinary differential equations (ODEs), we will gradually try to simulate higher-order equations, and Simulink provides an intuitive graphical modeling environment to simulate the behavior of the circuits by building the corresponding modules, which can efficiently validate the performance of the circuits designed by us in solving the ODEs of different orders, for example, to analyze the system's stability, accuracy, and responsiveness, and to identify potential problems and make optimization adjustments in time. Potential problems can be identified and optimized in a timely manner.

After the successful simulation, we will fabricate the printed circuit board (PCB) using KiCad, a powerful open-source electronic design automation software with complete functionality for schematic design and PCB layout and routing. With KiCad, we are able to draw accurate circuit schematics, rationally plan component layouts, ensure the correctness and reasonableness of electrical connections, and optimize wiring to reduce signal interference and improve system performance.

3 Ethics and Safety

In general, our project strictly follows the code of ethics of IEEE [9], and we will make detailed explanations in the following subsections.

3.1 Ethics Consideration

- **Intellectual Property**: Ensure that all the design concepts, circuit schematics, and algorithms used in the project are either original or properly cited if based on existing works. Avoid any form of plagiarism.

- **Open - Source and Collaboration**: Consider making the design of the analog computer open source, allowing other researchers and students to build upon and improve the work. This promotes collaboration and the sharing of knowledge within the scientific community.

3.2 Safety Consideration

- **Electrical Safety**: Since the analog computer is based on PCB boards with electrical components, proper insulation and grounding must be ensured. All electrical connections should be made according to safety standards to prevent electric shocks and short circuits.

- **Component Handling**: When handling components such as resistors, capacitors, and integrated circuits, proper anti-static measures should be taken to avoid damaging the components. Also, ensure that the components are rated for the voltage and current levels in the circuit to prevent overheating and potential fire hazards.

4 Conclusion

Though the idea of creating and applying analog computer to solve realistic problems has been raised since late 20th century, the application of analog computer only limits to solving certain types of theoretical equations. In this project, by building a general analog computer, we can further purpose a realistic idea of equipping analog computer in some real-world devices to process analog signals efficiently instead of transferring the signal into digital field and converting back which might cause large latency.

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