

ECE/CS 434 : Real-World Algorithms for IoT and Data Science :
Homework 1 Due 11 :59pm, Mon, Feb 16, 2026

Please answer each of the 5 questions on a new page, then scan and upload these pages onto Gradescope.

Problem 1 : State True/False with a 1 line justification [5x4=20 points]

(a) A is a $m \times n$ matrix with $m < n$. The null space $N(A)$ is always $\{\vec{0}\}$.

(b) The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, when left multiplied with a second matrix B (i.e. $B * A$), subtracts twice of the first column of B from the second column of B .

(c) For an orthonormal matrix Q (i.e., columns of Q are orthogonal to each other and the length of each column is 1), $Q^{-1} = Q^T$.

(d) If b_1, b_2, b_3 form the basis of a space, then $c_1b_1 + c_2b_2 + c_3b_3 = 0$ implies that all $c_1 = c_2 = c_3 = 0$.

(e) Matrix A has 6 columns, each column being a 10 dimensional vector. You are told that the dimension of $N(A^T)$ is 5. Then, $N(A)$ must be 1 and $\text{Rank}(A)$ must be 5.

Problem 2 : Symmetric Matrices [10+10=20 points]

(a) Prove that $A^T A$ is a symmetric matrix.

Hint : use the basic properties of transpose, as discussed in class.

(b) Prove that $\text{Rank}(AB) \leq \min\{\text{Rank}(A), \text{Rank}(B)\}$

Problem 3 : Column Spaces [10+10=20 points]

(a) Choose b which gives no solution and another b which gives infinitely many solutions. Your answer should show two values of b .

$$3x + 2y = 10 \tag{1}$$

$$6x + 4y = b \tag{2}$$

(b) Consider matrix $A_{m \times n}$. You are told $r = \text{Rank}(A)$ and $r < m$ and $r < n$. How many solutions are possible for the equation $Ax = b$? What is the dimensions of $N(A)$?

Problem 4 : Least Squares

[10 points]

Consider the following system of equations (called an over-determined system since there are more equations than unknowns) :

$$x - y = 2 \quad (3)$$

$$x + y = 4 \quad (4)$$

$$2x + y = 8 \quad (5)$$

How many solutions exist for the above system of equations ? If a solution exists find one, if not, determine the least squares solution for x and y .

Problem 5 : Eigenvalues and Eigenvectors [5+5+5+5+10=30 points]

(a) Prove that, for symmetric matrix A , eigenvalues of matrix $A^2 = (\text{Eigenvalue of matrix } A)^2$

(b) Prove that $\lambda(A - \sigma I) = (\lambda(A) - \sigma)$ where $\lambda(M)$ denotes the eigenvalues of matrix M , I is the identity matrix, and σ is an arbitrary constant.

(c) Given a matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$, calculate its eigen vectors e_1 , e_2 , and e_3 . Choose one eigen vector

e , plot e and $e' = A \cdot e$ in the 3D space. Consider another vector $x = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$. Plot x and $x' = A \cdot x$ in the 3D space.

(d) For the scenario above, write TRUE or FALSE :

- (i) if e and e' lie on the same line ;
- (ii) if x and x' lie on the same line.

(e) Consider the same matrix A as above. Use Python to plot the following points in 3D space.

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ -4 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Use another graph to plot their locations in a new 3D space where the basis of this new 3D space are the Eigen vectors of A . Please identify the points whose representation in the new space are unit vectors along x , y or z directions. Explain the relationship between these vectors and the Eigen vectors.