Minimum Variance Distortionless Response (MVDR) is an algorithm for the problem of multiple sources at different angles. The goal is to maximize the speech from one source, \( s_1 \), and suppress other sources, \( s_j \), at other angles.

An analogy can be drawn to a cocktail party: focus on Alice's speech.

Mathematically, the speech \( Y \) is modeled as

\[
Y = AS + n = \begin{bmatrix} a_1 & \ldots & a_d \end{bmatrix} \begin{bmatrix} s_1 \end{bmatrix} + n, \quad a_i = \begin{bmatrix} \vdots \end{bmatrix}
\]

Apply \( W^* \) to \( Y \):

\[
W^*Y = \hat{s}_1 + \frac{1}{\delta_{j=2}^d} s_j + \text{minimize}
\]

The formulation is

\[
\text{argmin}_W \quad \text{s.t.}
\]
Example of 3 Tx, 4 mics

\[
\begin{bmatrix}
  y'_1 \\
  y'_2 \\
  y'_3 \\
  y'_4
\end{bmatrix} =
\begin{bmatrix}
  a'_1 & a'_2 & a'_3 \\
  i & i & i \\
  a'_4 & a'_5 & a'_6
\end{bmatrix}
\begin{bmatrix}
  s_1 \\
  s_2 \\
  s_3
\end{bmatrix}
\]

Q: How could we combine signals from 3 antennas s.t. 1) \( s_1 \) is kept 2) \( s_2, s_3 \) are suppressed

\[ w_1 y_1 + w_2 y_2 + w_3 y_3 = \]

are small
Formulation:
\[
\text{arg min}_{w} \|w^* y^* w\| \quad \text{s.t.} \quad \|w^* a_i\| = 1
\]

Solution \( w = \)

\[w^* a_i = 1 = 1\]
**Problem:** Find $\theta_k$ for $k=4$ Txs w/ 6 mics

An algorithm

$\Rightarrow$ Delay and sum does not work well for

Why?

Microphone received space (complex, but here we draw real)

If are correlated

is large

Idea: Find the "" space

Lack color
### sub-space based AoA: MUSIC algorithm

\[ Y = A \bar{s} + \bar{n} \]
\[ Y Y^H = (A \bar{s} + \bar{n})(A \bar{s} + \bar{n})^H \]
\[ = (A \bar{s} + \bar{n})(s^H A^H + n^H) \]
\[ = A s^H A^H + A s \bar{n}^H + n s^H A^H + n n^H \]
\[ E[YY^H] = E[A s^H A^H + A s \bar{n}^H + n s^H A^H + n n^H] \]
\[ R_{Y} = A R_{s s} A^H + 0 + 0 + \sigma^2 I \]

#### Intuition:

\[ R_{xx} = A R_{s s} A^H + \sigma^2 I \]

\[ (R_{xx} - \sigma^2 I) \bar{e}_{d+1} = 0 \]

Both full rank
\[ A^H \bar{e}_{d+1} = 0 \]
“When you have eliminated the impossible, whatever remains, however improbable, must be the truth” — Sherlock Holmes
Problem: When sources are " ",

\[ S_2 = \alpha S_1 \]

\[ Y = \left[ \begin{array}{c} \left[ \begin{array}{c} I(t) \end{array} \right] + n \\ \vdots \end{array} \right] \]

= \quad + n

\[ \begin{cases} 1. & \text{(rank} 1 < 2) \\ 2. & \text{ Edn1} + \end{cases} \]

\[ P(O_2) \text{ may not be high!} \]
MUSIC spatial smoothing

⇒ Separate the antenna array into

\[ s \]

\[ Y_1 = \begin{bmatrix} \end{bmatrix} \]
\[ Y_2 = \begin{bmatrix} \end{bmatrix} \]
\[ + \) \[ Y_3 = \begin{bmatrix} \end{bmatrix} \]

\[ \sum_{g=1}^{G} Y_g = \begin{bmatrix} \end{bmatrix} \]

Note: \( Y_1^2 = Y_2^1 \)

⇒ Solve the correlated sources

⇒ Reduce the # of microphones in each group
AoA spectrum \( P(\theta_i) = \frac{1}{\| A_{\theta_i}^H e_{\text{noise}} \|_{L_1} } \)

\[ \begin{bmatrix} -A_{\theta_i} \end{bmatrix} \begin{bmatrix} e_{d1} & \cdots & e_{dN} \end{bmatrix} = \begin{bmatrix} \cdots \end{bmatrix} \]

But what happens with multipath?

Los path varies slowly while multipath varies fast.

Identify Los by observing stable AoA spectrum peak.

Apply Maximum Likelihood Estimation (MLE) for localization.