DSP : Discrete Fourier Transform (DFT) # 3
Let's do 3 simple examples.

1. DFT( \cos 2\pi f_m n t_s)
2. DFT( e^{j 2\pi f_m n t_s})
3. DFT( \sin 2\pi f_m n t_s)

\[
\begin{align*}
t_0 & \equiv x_0 = [e^{j0} \\
t_1 & \equiv x_1 = [e^{j2\pi f_m n t_s} \\
t_2 & \equiv x_2 = [e^{j2\pi f_m 2 t_s} \\
\vdots
\end{align*}
\]
Example 2: DFT \((x[n] = e^{j2\pi f_m n t s})\)

Just one stick rotating at \(f_m\) since it gives \(e^{j0.\pi}\)

Example 3: DFT \((x[n] = \sin 2\pi f_1 n t s)\)

\(n=0 \Rightarrow x_0 = \sin 0\)
\(n=1 \Rightarrow x_1 = \sin 2\pi f_1 t s\)
\(n=2 \Rightarrow x_2 = \sin 2\pi f_1 2t s\)

Slow stick \(\Rightarrow\) stick #1
Fast stick \(\Rightarrow\) stick #2

Like \(\cos 2\pi f_1 n t s\), you need 2 sticks to cancel out the complex values, but each stick now needs to start at different locations (or phases).
Translating to real-world frequencies

1 cycle in $N$ time steps
1 cycle in $N \times \frac{t_s}{s}$ seconds
$\times$ cycles in 1 second

\[ \frac{1}{t_s} = \text{freq.} = f_s \]
\[ x = \frac{1}{N t_s} = \frac{1}{N} \cdot \frac{1}{t_s} = \frac{f_s}{N} \]

- Slowest frequency = \( \frac{f_s}{N} \)

\( f_s \) \Rightarrow \text{Called Fundamental freq.}

Faster freq. \( \Rightarrow \) \( 2 \frac{f_s}{N}, 3 \frac{f_s}{N}, 4 \frac{f_s}{N}, \ldots, (N-1) \frac{f_s}{N} \)

For large $N$, \( (N-1) \approx N \),

\[ \text{\textbullet\ max freq} = f_s \]

Bandwidth = \( f_s \).
Consider sampling your voice signal at 8000 Hz and taking a 1000 point FFT (i.e., $f_s = 8000 \text{ Hz}$, $N = 1000$).

Thus, when analyzing a given signal, we have 2 knobs we can control:

- **Sampling freq $f_s$**
- **No. of samples you take in your FFT $= N$**

Large $f_s$ means we can see up to large freq components in the signal.

Large $N$ means we have finer freq. resolution (or more freq. bins within $[0, f_s]$ band).

Width of bin = $\frac{f_s}{N}$.
f₁ = 1

BEWARE OF INCOMPLETE CYCLES.

Cos 2πf₁x
f₁ = 2

what is the FFT mag. spectrum for these signals?

Now, what is the FFT mag. spectrum of this

Think of this X₃ signal as

Thus, DFT(X₃) ≠ DFT(X₂) since the DFT influences results.

In fact:

is DFT(X₄) ≠ DFT(X₂) ≠ DFT(X₁)
because 1½ cos(·) cycles cannot be made up by integer no. of rotations of cos(f₁) & cos(fₙ₋₁)

Many different rotations are needed =)

Think of this as
REAL SPECTROGRAM: Voice Signal

- FFT of first window
- FFT of sliding window

Time Frequency bin:

Shows various freq. components at this time slice.

Spectrogram

- Time domain signal

Shows how a specific freq. comes and goes over time.

$N=1000$

$\text{FFT} \in \mathbb{C}$