# ECE/CS 434 : Mobile Computing Algorithms and Applications : Homework 3 : Due 11 :59pm, Mon, Mar 27, 2023

## Problem 1 : DUET [3x2=6 points]

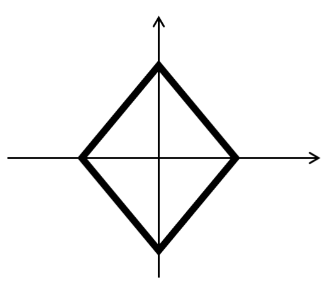
1. True or false, and provide one sentence explanation : DUET performs better when the angles of arrival of the 2 sources have greater angular separation.
2. Complete the sentence : DUET does not work when 2 music sources are playing together because ...

## Problem 2 : GPS [3 points]

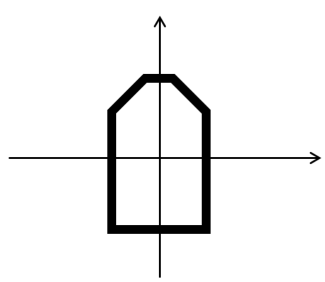
1. True or false, and provide one sentence explanation : If satellites were not synchronized, it would be impossible to solve the localization equation in GPS (given that local and satellite clocks are unsynchronized).

## Problem 3 : PCA / SVD [2x6+5+3x4+3=32 points]

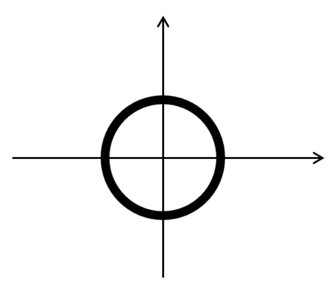
1. For each of the 2D data visualization below, identify whether X and Y are (1) correlated or not, and (2) dependent or not. For (a) to (e), assume the data points are uniformly distributed within the shape boundary. For (f), assume the distribution is a 2D joint gaussian distribution.



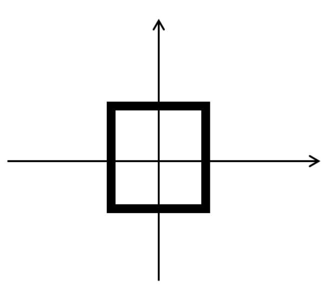
a



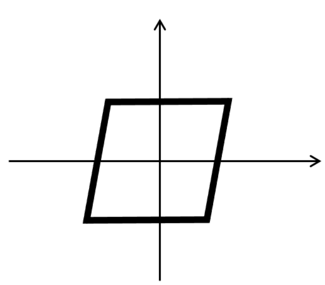
b



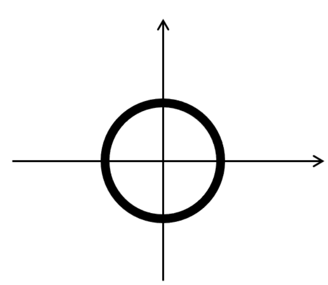
c



d



e



f

2. Prove that projecting the data onto the principle components makes the data uncorrelated.
3. T/F and provide one sentence explanation :
   (a) For Eigen decomposition $A = S\Lambda S^{-1}$ , the eigenvectors in $S$ all lie in $C(A)$
   (b) Assume we have a thin matrix $A_{m\times n}$ $(m > n)$, whose columns are all linearly independent. In the SVD decomposition $A = U\Sigma V^T$, all $m$ eigen vectors in $U$ lie in $C(A)$
   (c) In the same scenario above, some Eigen vectors in $U$ must lie in the null space $N(A)$
   (d) In the same scenario above, all the Eigen vectors in $U$ must be perpendicular to each other
4. Think and write a problem from the real world whose solution requires either SVD or PCA. You cannot use the 2 problems discussed in class (namely recommendation engines and data compression). Use no more than 20 words.

## Problem 4 : Gradient Descent [4x5=20 points]

1. Consider $f(x) = x_1^2 + x_2^2 - 3x_1x_2 + 2x_1 + 3x_2 + 7$. Compute $\nabla f(x)$, $\nabla^2 f(x)$.
2. Is $f(x)$ convex ? Prove using equations.
3. Prove that the sum of two convex functions is a convex function.
4. Consider $g(x) = x_1^2 + 2x_2^2 + 5$. Using a step size of 0.2, show the sequence of $x_k$ for the steepest gradient descent (SGD) algorithm.
5. Is it possible that SGD would not converge with $g(x)$ even if a finite step size is used ? Explain briefly.

## Problem 5 : Maximum Likelihood Estimation [5 points]

1. Given a random variable $X$ with sampled measurements as [0,1,1,2,2,3,3,3,4,4,5,5,6], using MLE, estimate the mean and variance of the Gaussian distribution. Please solve it using pen and paper.

## Problem 6 : Beamforming [5x5=25 points]

1. Derive the equation for the radiation pattern for $N$ sensors with $D$ separation, for a frequency $F$. Assume the wavelength is $\lambda$.
2. Visualize the radiation patterns for $N = 2, 4, 8, 16, 32$ for a fixed $D = \lambda/2$.
3. Visualize the radiation patterns when $F$ increases while $D$ remains unchanged.
4. Explain why $D = \lambda/2$ is necessary ; what happens when D grows larger.
5. What happens when $D$ is made smaller ?

## Problem 7 : FMCW [5+4=9 points]

1. Show that a random noise $X$, where each sample is drawn independently from a standard normal distribution, has autocorrelation $Rx(T) = 0$ except for $T = 0$.
2. In your research group, you are proposing the FMCW method to estimate the distance of a human in a room using wireless radios. Your colleague argues this will not work since echoes of your signal (that bounce off walls and ceilings) will all come back to the FMCW radio and that will pollute your ability to estimate the distance to the user. In no more than 20 words, argue in favor or against this criticism.