

ECE 431 Electric Machinery

NAME: Solution

Test #2 April 6, 2016

You may use a one-sided sheet of *your own* hand-written notes as reference.

Please do all work on this booklet. Label any solutions that are written on backs of pages or on this spare sheet.

Q1. (25 points):

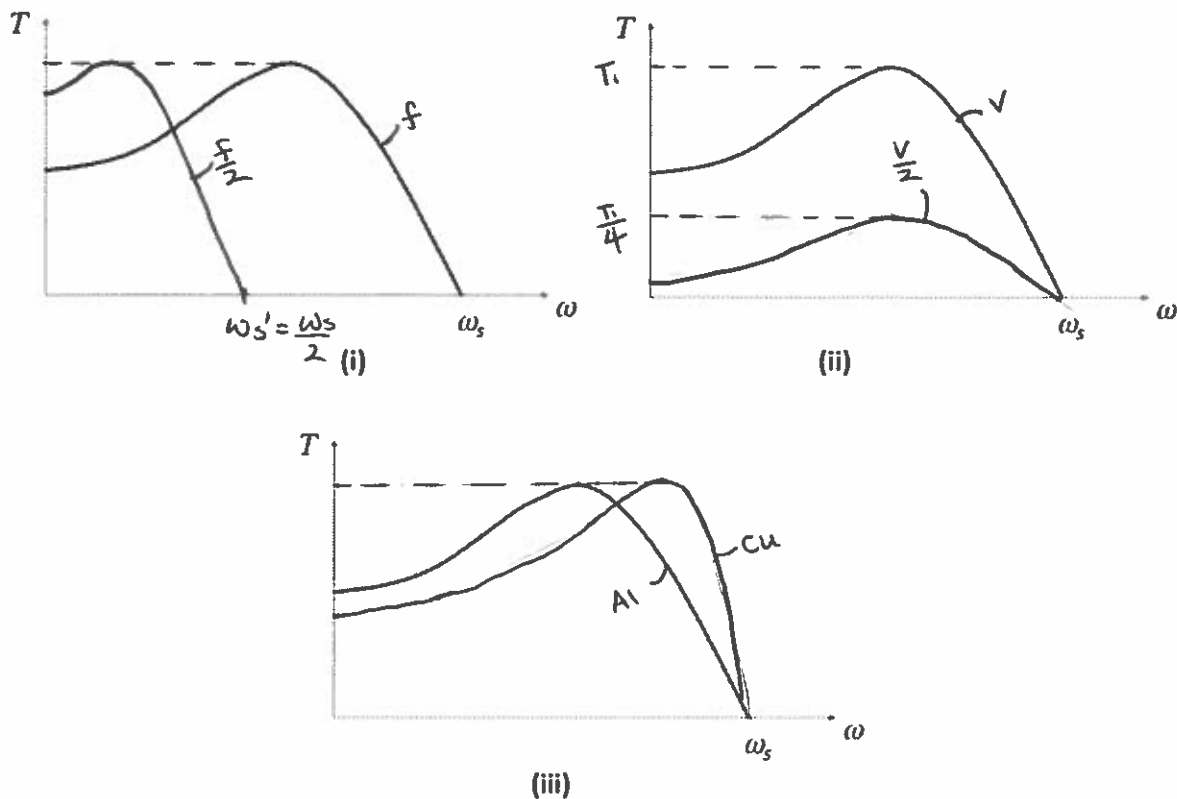
(a) List three means by which you can control the speed of an induction motor and give one disadvantage of each.

- Rotor resistance : need for slip rings
- Frequency : Need for external inverter (VFD)
- Input voltage : Lowers torque capabilities
- Pole number : Mechanical challenges

(b) List two ways you can increase the starting torque of an induction motor

- Increase rotor resistance
- Variable frequency control

(c) Superimpose on the baseline induction motor torque-speed curve shown the modified curve if (i) the drive frequency is halved, (ii) the drive voltage is halved. (iii) the aluminum cage is replaced with a copper cage of same dimensions. Point out salient features.



Q2. (25 points):

A balanced, symmetrical, wye-connected, three-phase, 60 Hz, 4-pole, 480 V (line- to -line), 10kW induction machine has $X_1 = X_2' = 1.5 \text{ Ohms}$, $R_1 = 0.4 \text{ Ohms}$, $R_2' = 0.35 \text{ Ohms}$. Ignore X_m and R_c for this question (i.e. assume the magnetizing impedance is very large).

This motor is used to drive a load whose torque as a function of mechanical angular velocity is given by:

$$T_{\text{load}} = (36 + 0.1 \omega_m) \text{ Nm}$$

a) Will the motor be able to start? ($N_m = 0$, $s = 1$ at start)

$$\begin{aligned} T_{\text{start}} &= \frac{3P}{2\omega_e} \frac{|V_1|^2 R_r'}{[(R_s + R_r')^2 + (X_s + X_r')^2]} \\ &= \frac{(3)(4)}{2(2\pi \cdot 60)} \frac{\left(\frac{480}{\sqrt{3}}\right)^2 (0.35)}{[0.75^2 + 3^2]} = \underline{44.74 \text{ Nm}} \end{aligned}$$

$$T_{\text{load at start}} = \underline{36 \text{ Nm}}$$

\Rightarrow The motor will be able to start.

b) At what speed (rpm) and slip, s , would the motor run with this load? (Hint: use small slip approximation for torque to simplify your calculations)

$$T_e = \frac{3|V_1|^2}{\omega_s R_r'} s = 36 + 0.1 \omega_s (1-s) \quad \text{where } \omega_s : \text{mechanical synchronous speed [rad/s]}$$

$$\begin{aligned} s &= 0.015 \\ \omega_m &= 1773 \text{ RPM} \end{aligned}$$

c) Compute the line current, and power factor of the machine at this load?

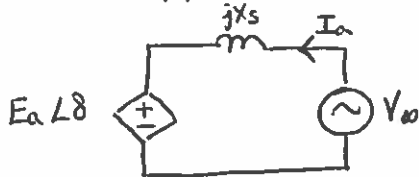
$$\begin{aligned} I_L &= \frac{|V_1|}{R_s + \frac{R_r'}{s} + j(X_s + X_r')} = \frac{480/\sqrt{3}}{0.4 + \frac{0.35}{0.015} + j(3)} = 11.58 \angle -7.2^\circ \\ &\Rightarrow \text{PF} = \cos(7.2) = 0.99 \end{aligned}$$

$$\begin{aligned} |I_L| &= 11.58 \text{ A} \\ \text{PF} &= 0.99 \text{ lagging} \end{aligned}$$

Q3. (25 points):

A 480 Volt (line to line) Y-connected, 4-pole, 60Hz synchronous motor is drawing 40 Amps at 0.9 power factor lagging from a 480 Volt (line to line) 'infinite' grid. The field current under these conditions is 6 Amps. The synchronous reactance of the motor is 8 Ohms. Assume a linear open-circuit characteristic.

(a) Find the internal voltage and torque angle for this condition



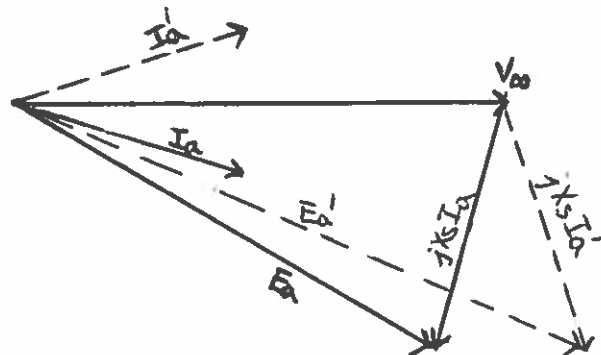
$$E_a \angle \delta = V_\infty - jX_s \bar{I}_a \quad \text{where } \bar{I}_a = 40 \angle -\cos^{-1}(0.9) = 40 \angle -25.84^\circ$$

$$= \frac{480}{\sqrt{3}} - j(8)(40 \angle -25.84^\circ)$$

$$= 319.21 \angle -64.54^\circ \text{ V}$$

$|E_a| = 319.21 \text{ V}$
 $\delta = -64.54^\circ$

(b) Draw the phasor diagram for this condition. On the same axis, use dashed lines to show the phasors for 0.9 leading operation while consuming the same amount of real power.



(c) Compute the field current required to make the motor operate at 0.9 power factor leading with the same real power?

$$\bar{I}_a' = 40 \angle 25.84^\circ$$

$$\bar{E}_a' = \frac{480}{\sqrt{3}} - j(8)(40 \angle 25.84^\circ)$$

$$= 506.46 \angle -34.66^\circ \quad \rightarrow \quad I_f' = \frac{E'}{E} \cdot I_f = 6 \frac{506.46}{319.21} = \boxed{9.52 \text{ A}}$$

Q4 (25 points)

A salient-pole synchronous generator has direct-axis and quadrature-axis reactances of 1.2 and 0.8 per unit, respectively. Assume armature resistance is negligible. The generator is delivering rated current at 0.8 power factor lagging, with rated terminal voltage.

(a) Compute the load angle

$$\begin{aligned}\bar{E}'_a &= \bar{V}_a + jX_q \bar{I}_a \quad \text{where } I_a = 1 \angle -\cos^{-1}(0.8) = 1 \angle -36.9^\circ \\ &= 1 + j(0.8)(1 \angle -36.9^\circ) \\ &= 1.6 \angle 23.3^\circ\end{aligned}$$

$$\Rightarrow \boxed{\delta = 23.3^\circ}$$

(b) Compute the direct and quadrature axis currents, in per-unit

$$I_d = 1 \sin(\delta - \theta) = \sin(23.3^\circ - (-36.9^\circ)) = 0.86$$

$$I_q = 1 \cos(\delta - \theta) = \cos(23.3^\circ - (-36.9^\circ)) = 0.5$$

$$\boxed{\begin{aligned}I_d &= 0.86 \text{ pu} \\ I_q &= 0.5 \text{ pu}\end{aligned}}$$

(c) Compute the per-unit internal voltage, E_a

$$\begin{aligned}|E_a| &= V_q + I_d X_d \\ &= 1 \cos \delta + (0.86)(1.2) = 1.95\end{aligned}$$

$$\boxed{|E_a| = 1.95 \text{ pu}}$$

(d) Draw the phasor diagram and identify all voltages and currents

