ECE 431 Electric Machinery

NAME: SOLUTION

Test 2. April 9, 2014

Closed book, but you may refer to your own handwritten notes, 50 min.

Problem points (out of 100) are indicated. Work all problems in exam booklets.

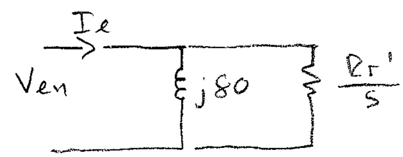
Q1. (20 points):

A balanced, symmetrical, 3-phase, Y-connected, 60 Hz, 6-pole, 408 Volt (line-line) induction motor has negligible stator resistance, and negligible stator and rotor leakage reactance. The magnetizing reactance referred to the stator is 80 Ohm per phase. You may ignore core losses and mechanical losses. The motor delivers a load torque of 13 NM at a speed of 1080 RPM.

(a) What is the frequency of the rotor currents at this operating point?

$$(1200 - 1080) \cdot \frac{P}{2} \times \frac{1}{60} = \boxed{6\ Hz}$$

(b) What is the magnitude of the line current?



$$S = \frac{1200 - 1080}{1200} = 0.1$$

$$I_e = V_{en} \Big/ \left(\frac{j80 \cdot \frac{R_r'}{s}}{j80 + \frac{R_r'}{s}} \right) = 3.74 \angle -51.9^\circ\ A$$

$$P_{ag1\phi} = \frac{V_{en}^2}{\frac{R_r'}{s}}$$

$$\boxed{|I_e| = 3.74\ A}$$

$$P_{m1\phi} = (1-s) P_{ag1\phi} = s(1-s) \frac{V_{en}^2}{R_r'}$$

$$= \frac{T_m}{3} \cdot (1-s)\omega_s$$

$$\hookrightarrow R_r' = \frac{3s V_{en}^2}{T_m \omega_s} = \frac{3(0.1)\left(\frac{408}{\sqrt{3}}\right)^2}{13\left(1200 \cdot \frac{2\pi}{60}\right)} = \underline{10.19\,\Omega}$$

(c) What is the power factor of the machine at this load?

Assuming $\measuredangle V_{en} = 0^\circ$

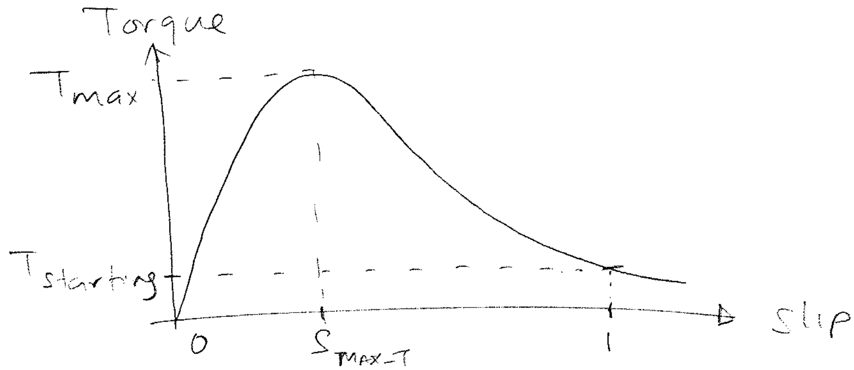
$$pf = \cos(\measuredangle V_{en} - \measuredangle I_e)$$

$$= \cos(0 + 51.9^\circ)$$

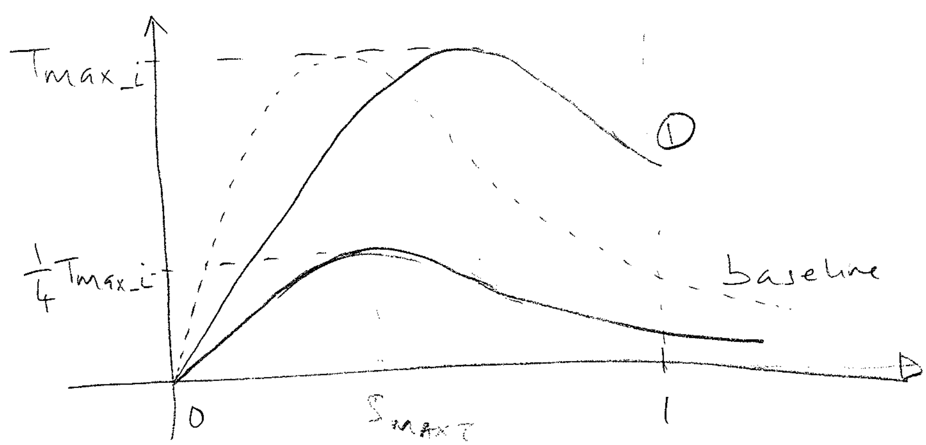
$$= \boxed{0.617\ lag}$$

Q2. (15 points)

(a) Draw the torque-slip curve of an induction motor and label the axes and key features of the curve.



(b) Draw a baseline torque-slip curve and superimpose on it the modified curve for, (i) the drive frequency is halved and (ii) the drive voltage is halved.



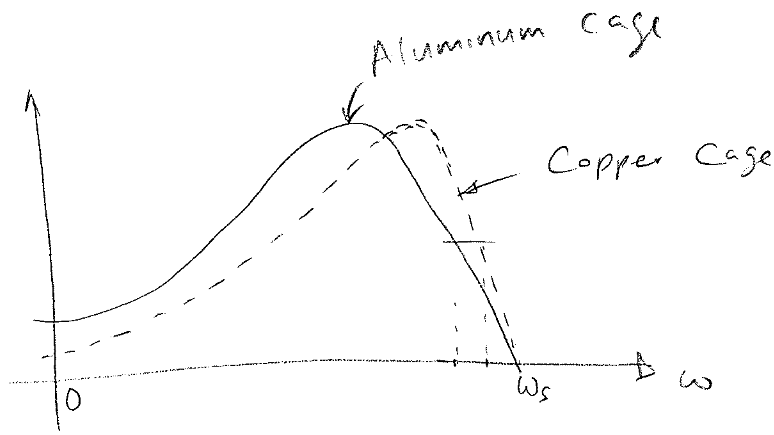
(c) List three means by which you can control the speed of an induction motor.

Rotor resistance
Stator frequency
Number of poles
Stator voltage
Rotor frequency

(d) What are all the changes in an induction motor performance you would expect if the rotor cage material is changed from Aluminum (resistivity=3 x 10^{-8} Ohm-m) to Copper (resistivity=1.8 x 10^{-8} Ohm-m)?

Rotor resistance, r_2, reduces by ~40%.

⇒ Operating slip will reduce
Efficiency improves
Starting torque reduces



Q3. (20 points):

A 400kVA, 3kV L-L, 3-phase, 60 Hz, 4-pole, Y-connected, synchronous **motor** running at 1800 pm gives the following test data:

DC Resistance test across two line terminals gives 27A for 5V DC.

Open Circuit: If = 36A, Voltage = 3 kV L-L.

Short Circuit: If = 30A, Ia = 77 A

a) Find the short circuit ratio

$$SCR = \frac{I_{f,NL}}{I_{f,SC}} = \frac{36\ A}{30\ A} = \boxed{1.2}$$

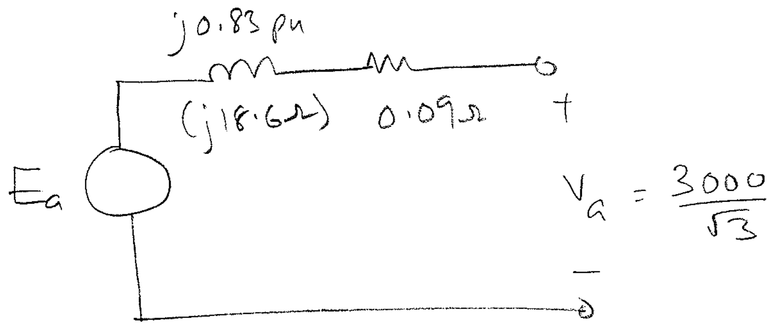
b) What is the saturated value of the synchronous reactance in per units and in Ohms per phase?

$$X_{s,\ p.u.} = \frac{1}{SCR} = \frac{1}{1.2} = \boxed{0.83\ p.u.}$$

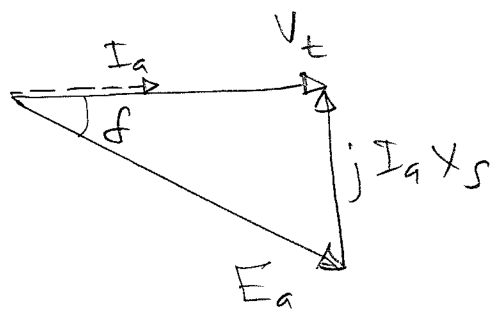
$$Z_{base} = \frac{\left(\frac{3kV}{\sqrt{3}}\right)^2}{\frac{400\ kVA}{3}} = 22.5\ \Omega$$

$$\therefore X_s = 0.83 \times 22.5\ \Omega = \boxed{18.675\ \Omega}$$

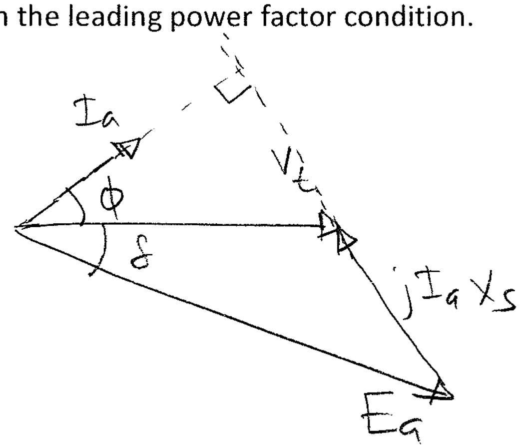
c) Draw the per-phase equivalent circuit for the motor and label all elements and enter all known values.



d) The motor is operated at rated voltage and current at unity power factor. Draw the phasor diagram for this case.



e) Re-draw the phasor diagram the leading power factor condition.



Q4. (20 points)

A synchronous generator with a synchronous reactance of 0.5 p.u. is connected to a 60Hz infinite bus at 1pu voltage. The generator is delivering 0.85 per unit power to the bus at steady state, with a load angle (δ) of 15°. A circuit breaker now opens isolating the generator from the infinite bus (no power supplied to the grid).

a) Determine the p.u. internal voltage at the initial condition.

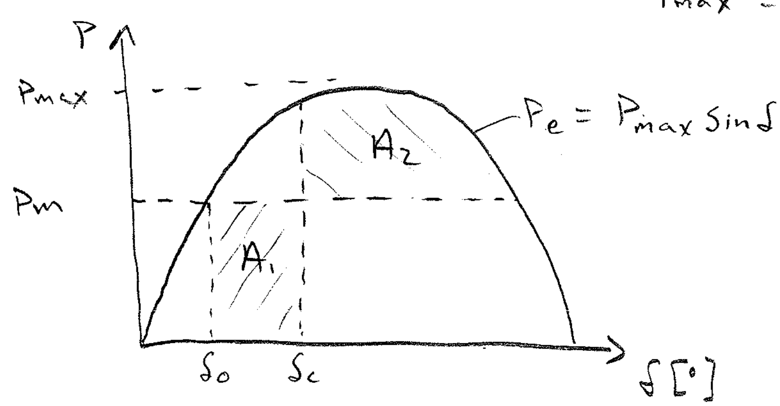
$$P_{3\phi,p.u.} = \frac{|V_{an}||E_{af}|}{X_s} \sin\delta$$

$$0.85 = \frac{(1)\,(E_{af})\,\sin(15°)}{0.5}$$

$$\boxed{E_{af} = 1.642 \text{ p.u.}}$$

b) Draw the power curve for the generator and mark the areas to be used in the equal area calculation.

$$P_{max} = \frac{(1)(1.642)}{0.5} = 3.284 \text{ p.u.}$$



$A_2 > A_1$ for stability

c) Calculate the critical clearing angle.

$$A_1 = \int_{\delta_0}^{\delta_c} P_m \, d\delta$$

$$= P_m \left(\delta_c - \delta_0\right)$$

$$A_2 = \int_{\delta_c}^{\pi-\delta_0} (P_e - P_m) \, d\delta$$

$$= \int_{\delta_c}^{\pi-\delta_0} (P_{max} \sin\delta - P_m) \, d\delta$$

$$= -P_{max} \cos(\pi - \delta_0) - P_m(\pi - \delta_0)$$
$$+ P_{max} \cos(\delta_c) + P_m(\delta_c)$$

for $A_1 = A_2$

$P_{max} = 3.284$ p.u.

$P_m = 0.85$ p.u.

$\delta_0 = \pi/12$

$$A_1 = A_2$$

$$0.85\left(\delta_c - \frac{\pi}{12}\right) = -3.284 \cos\left(\pi - \frac{\pi}{12}\right) - 0.85\left(\pi - \frac{\pi}{12}\right)$$
$$+ 3.28 \cos(\delta_c) + 0.85(\delta_c)$$

$$\delta_c = 1.862 = \boxed{106.7^\circ}$$

Q5. (25 points)

a) What are some fault tolerance challenges with PM motors relative to wound field synchronous machines? Discuss possible ways to address these.

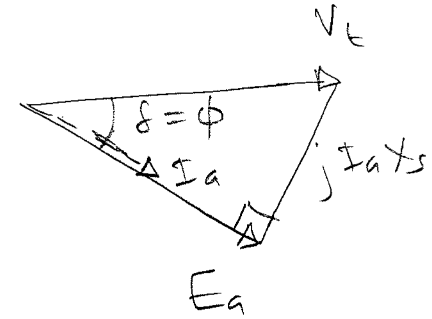
— High short circuit currents because of relatively low X_s
↳ design with higher reactance
buck flux with I_d if possible

— No way to control / turn off field.
↳ flux weakening / mechanically reduce machine speed

— Large back-emf at high speeds could lead to excessive voltages during open circuit → field weakening, auxiliary wdg.

b) A 6-pole, 3ph surface mount PM motor is rated 75kVA, 480V_{L-L}, 1200 rpm. The motor is observed to have a line-to-line internal voltage of 460 V and a synchronous reactance of 0.4 Ohms at rated speed. The motor is supplied from a current source converter which implements a V/Hz control, and limits terminal voltage and current to rated values.

(i) Draw the phasor diagram showing voltages and current phasors to obtain the maximum torque and power assuming terminal voltage limit is not hit.



(ii) Calculate the maximum torque and power the motor can supply at rated speed and rated current.

Rated line current:

$$I_\ell = \frac{75\,kVA}{3} \Big/ \frac{480V}{\sqrt{3}} = 90.21\,A$$

Max power motor can supply

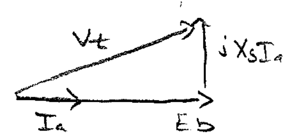
$$P_{max} = E_b I_\ell$$

$$= 3 \cdot \frac{460V}{\sqrt{3}} \cdot 90.21A$$

$$\boxed{P_{max} = 71.87\,kW}$$

$$T_{max} = \frac{P_{max}}{\omega_m} = \frac{71.87\,kW}{1200 \times \frac{2\pi}{60}} = \boxed{571.96\,Nm}$$

Check terminal voltage



$$|V_t| = \sqrt{\left(\frac{460}{\sqrt{3}}\right)^2 + (90.21^2 + 0.4)^2}$$

$$= \underline{268\,V_{\ell-n}}$$

$V_{t\ell\ell} = 464.23V < V_{rated}$ ✓

No voltage violation

(iii) The motor is now operated at a higher speed leading to too large an internal voltage that flux weakening by the drive is required. Re-draw the phasor diagram showing voltages and current phasors under this condition.