$$g[n] = w[n]\,h[n]$$

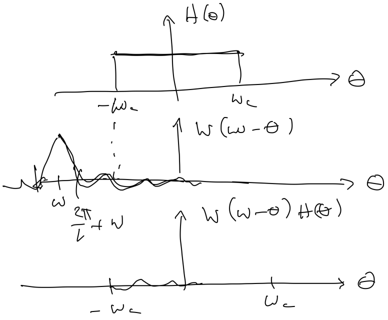
$$\rightarrow h[n] = \left(\frac{\omega_c}{\pi}\right)\text{sinc}(\omega_c n)$$

$$\rightarrow w[n] = \begin{cases} 1 & -M \le n \le M \\ 0 & \text{else} \end{cases}$$

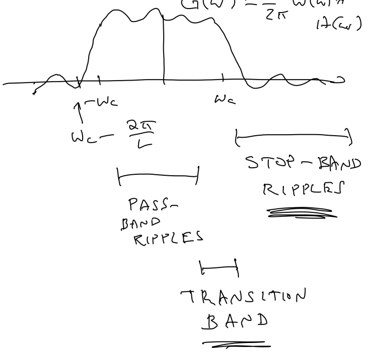
$$\rightarrow H(\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \text{else} \end{cases}$$

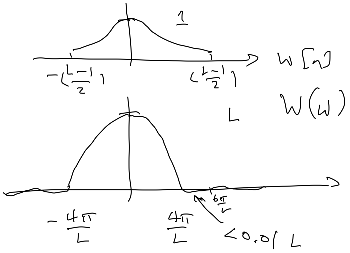
$$w[n] = \begin{cases} 1 & -\left(\frac{L-1}{2}\right) \le n \le \left(\frac{L-1}{2}\right) \\ 0 & \text{else} \end{cases}$$

$$\rightarrow W(\omega) = \frac{\sin(\omega L/2)}{\sin(\omega/2)}$$



$$G(\omega) = \frac{1}{2\pi} W(\omega) * H(\omega)$$



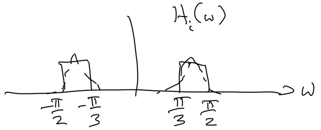


EXAMPLE

$h[n]$ $\omega_2 = \frac{\pi}{2}$

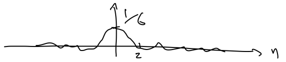
$\omega_1 = \frac{\pi}{3}$

$L = 33$ SAMPLES



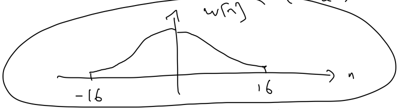
$$h_i[n] = \left(\frac{\omega_2}{\pi}\right)\mathrm{sinc}(\omega_2 n) - \left(\frac{\omega_1}{\pi}\right)\mathrm{sinc}(\omega_1 n)$$

$$= \frac{1}{2}\,\mathrm{sinc}\left(\frac{\pi n}{2}\right) - \frac{1}{3}\,\mathrm{sinc}\left(\frac{\pi n}{3}\right)$$

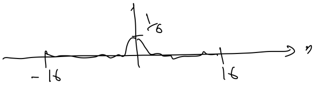


$$h[n] = w[n]\, h_i[n]$$

$$w[n] = 0.54 + 0.46\cos\left(\frac{2\pi n}{32}\right)$$



$$h[n] = \left(0.54 + 0.46\cos\left(\frac{2\pi n}{32}\right)\right) \times \left(\frac{1}{2}\,\mathrm{sinc}\left(\frac{\pi n}{2}\right) - \frac{1}{3}\,\mathrm{sinc}\left(\frac{\pi n}{3}\right)\right)$$



$$E_{q|x}[\ln p(q,X)]$$

$$= -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{N}\gamma_t(i)\left[\cancel{\ln|\Sigma_{1i}|} + (x_t-\mu_i)^T\Sigma_{1i}^{-1}(x_t-\mu_i)\right]$$

$$\frac{\partial E_{q|x}[\ln p(q,X)]}{\partial \mu_j}$$

$$= -\cancel{\frac{1}{2}}\sum_{t=1}^{T}\gamma_t(j)\,\cancel{(-2)}\,\Sigma_{1j}^{-1}(x_t-\mu_j)$$

$$= \sum_{t=1}^{T}\gamma_t(j)\,\Sigma_{1j}^{-1}\underset{\overset{\uparrow}{T}}{(x_t-\mu_j)} = 0$$

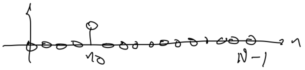
$$\sum_{t=1}^{T}\gamma_t(j)\,\Sigma_{1j}^{-1}x_t = \sum_{t=1}^{T}\gamma_t(j)\,\Sigma_{1j}^{-1}\mu_j$$

$$\sum_{t=1}^{T}\gamma_t(j)\,x_t = \sum_{t=1}^{T}\gamma_t(j)\,\mu_j = \mu_j\sum_{t=1}^{T}\gamma_t(j)$$

$$\frac{\sum_{t=1}^{T}\gamma_t(j)\,x_t}{\sum_{t=1}^{T}\gamma_t(j)} = \underline{\underline{\mu_j}}$$

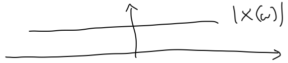
10/23/2023

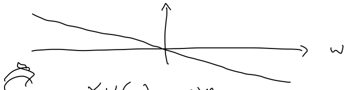
$$x[n] = \delta[n-n_0]$$



$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k n}{N}}$$

$$= e^{-j\frac{2\pi k n_0}{N}}$$

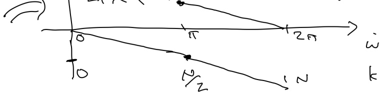




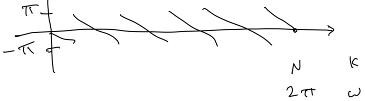
$$\measuredangle X(\omega) = -\omega n_0$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = e^{-j\omega n_0}$$

$$\measuredangle X(\omega) = -\omega n_0 = -\frac{2\pi k n_0}{N}$$



$$\measuredangle X(\omega) = \measuredangle X[k] = -\frac{2\pi k n_0}{N}$$



CONJUGATE SYMMETRY

$$X(\omega) = X^*(-\omega)$$

$$X[k] = \underline{X^*[-k]} = \underline{X^*[N-k]}$$

$$X(\omega) = e^{-j\omega n_0}$$

$$X(-\omega) = e^{-j(-\omega)n_0}$$

$$= e^{j\omega n_0} = X^*(\omega)$$

$$X[k] = e^{-j\frac{2\pi k}{N}n_0} \Leftarrow$$

$$X[N-k] = e^{-j\frac{2\pi(N-k)}{N}n_0} \Leftarrow$$

$$= e^{-j\frac{2\pi N}{N}n_0} e^{-j\frac{2\pi(-k)}{N}n_0}$$

$$= \left(e^{-j2\pi}\right)^{\frac{N n_0}{N}} e^{j\frac{2\pi k n}{N}}$$

INTEGER

1

$$= (X[k])^*$$

$$X[-k] = e^{-j\frac{2\pi(-k)}{N}n_0}$$

$$= e^{j\frac{2\pi k n}{N}} = X^*[k]$$

Practice Exam

$\boxed{1}$ $P(q_t = G \mid q_{t-1})$ $P(q_t = B \mid q_{t-1})$

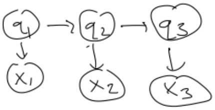
q_{t-1}	G	0.7	0.3
	B	0.4	0.6

$\mu_G = 50, \quad \sigma_G = 20$

$\mu_B = 10, \quad \sigma_B = 20$

$X = [10, 20, 30]$

$Pr(X \mid q_1 = B) = Pr(x_1 = 10, x_2 = 20, x_3 = 30 \mid q_1 = B)$



$$Pr(X \mid q_1 = B) = \sum_{q_2}\sum_{q_3} Pr(q_2, q_3, x_1, x_2, x_3 \mid q_1)$$

$$= \sum_{q_2}\sum_{q_3} Pr(x_1 \mid q_1 = B)\, Pr(q_2 \mid q_1 = B)\, Pr(x_2 \mid q_2)\, Pr(q_3 \mid q_2)\, Pr(x_3 \mid q_3)$$

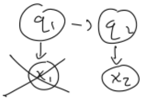
$$= \sum_{q_2}\sum_{q_3} N(10; 10, 20)\, a_{B,q_2}\, N(20; \mu_{q_2}, 20)\, a_{q_2,q_3}\, N(30; \mu_{q_3}, 20)$$

BBB, BBG, BGB, BGG

$$= N(10; 10, 20)\, 0.6\, N(20; 10, 20)\, 0.6\, N(30; 10, 20)$$
$$+ N(10; 10, 20)\, 0.6\, N(20; 10, 20)\, 0.4\, N(30; 50, 20)$$
$$+ N(10; 10, 20)\, 0.4\, N(20; 50, 20)\, 0.3\, N(30; 10, 20)$$
$$+ N(10; 10, 20)\, 0.4\, N(20; 50, 20)\, 0.7\, N(30; 50, 20)$$

where

$$N(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$



$$Pr(q_2 \mid q_1) = \sum_{x_1}\sum_{x_2} Pr(x_1 \mid q_1)\, Pr(q_2 \mid q_1)\, Pr(x_2 \mid q_2)$$

$$= Pr(q_2 \mid q_1) = a_{q_1,q_2}$$

$\boxed{2}$

INITIALIZE FIND $\tilde{\alpha}_1(i)$, g_1

ITERATE FIND $\tilde{\alpha}_t(i)$, g_t

TERMINATE FIND $P(X)$

$\tilde{\alpha}_t(i) = Pr(q_t = i \mid x_1, \ldots, x_t)$

$g_t = Pr(x_t \mid x_1, \ldots, x_{t-1})$

INITIALIZE

$\alpha_1(i) = Pr(q_1 = i \mid x_1)$

$$= \frac{\Pr(q_1 = i, x_1)}{\sum_j \Pr(q_1 = j, x_1)}$$

$$= \frac{\Pr(q_1 = i)\Pr(x_1 \mid q_1 = i)}{\sum_j \text{NUMERATOR}}$$

$$= \frac{\pi_i b_i(x_1)}{\sum_j \pi_j b_j(x_1)}$$

$$g_1 = \Pr(x_1 \mid \text{——}) = \sum_j \pi_j b_j(x_1)$$

ITERATE

$$g_t = \Pr(x_t \mid x_1, \dots, x_{t-1})$$

$$= \sum_j \Pr(q_t = j, x_t \mid x_1, \dots, x_{t-1})$$

$$= \sum_j \sum_i P(q_{t-1} = i \mid x_1, \dots, x_{t-1}) \Pr(q_t = j \mid q_{t-1} = i) \Pr(x_t \mid q_t = j)$$

$$g_t = \sum_j \sum_i \tilde{\alpha}_{t-1}(i)\, a_{i,j}\, b_j(x_t)$$

$$\tilde{\alpha}_t(i) = \Pr(q_t = i \mid x_1, \dots, x_t)$$

$$= \frac{\Pr(q_t = i, x_t \mid x_1, \dots, x_{t-1})}{\left(\sum_i \text{NUMERATOR}\right] = g_t}$$

$$= \frac{\sum_j \Pr(q_{t-1} = j \mid x_1, \dots, x_{t-1}) \Pr(q_t = i \mid q_{t-1} = j) P(x_t = q_t = i)}{\sum \text{NUMERATOR}}$$

$$= \frac{\sum_j \tilde{\alpha}_{t-1}(j)\, a_{j,i}\, b_i(x_t)}{\sum_i \sum_j \tilde{\alpha}_{t-1}(j)\, a_{j,i}\, b_i(x_t)}$$

TERMINATION

$$\Pr(x) = \Pr(x_1, \dots, x_T)$$

$$= \Pr(x_T \mid x_1, \dots, x_{T-1}) P(x_{T-1} \mid x_1, \dots, x_{T-2}) \cdots$$

$$= g_T\, g_{T-1}\, g_{T-2} \cdots g_1$$

$$= \prod_{t=1}^{T} g_t$$

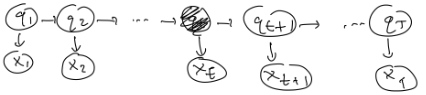
b) $\beta_t(i) = \Pr(x_{t+1}, \dots, x_T \mid q_t = i)$

$\tilde{\alpha}_t(i)\beta_t(i) = \Pr(f \mid g)$ FOR SOME f, g

$\tilde{\alpha}_t(i) = \Pr(q_t = i \mid x_1, \dots, x_t)$

$B: q_t = i$

$\therefore \tilde{\alpha}_t(i)\beta_t(i) = P(\underline{q_t = i} \mid \underline{x_1, \dots,} x_t)\, P(\underline{x_{t+1}, \dots, x_T} \mid \underline{q_t = i}) = \Pr(B \mid A)\Pr(C \mid B)$



$$\boxed{\Pr(A, B, C) = \Pr(C \mid A, B)\Pr(B \mid A)\Pr(A)}$$

$$\boxed{= \Pr(C \mid B)\Pr(B \mid A)\Pr(A)}$$

$$\tilde{\alpha}_t(i)\beta_t(i) = \Pr(q_t = i, x_{t+1}, \dots, x_T \mid x_1, \dots, x_t)$$

$$f = \{q_t, x_{t+1}, \dots, x_T\}$$

$g = \{x_1, \dots, x_t\}$

3

$$[x_1, x_2, x_3, x_4] = \begin{bmatrix} 120 & 140 & 140 & 120 \\ 0.3 & 0.3 & 0.5 & 0.5 \end{bmatrix}$$

a Estimate Σ_1 ⓑ FIND $d_{\Sigma_1}(x_5, x_6)$

$$\sigma_1^2 = \frac{1}{M-1} \sum_{m=1}^{M} (x_{1,m} - \mu_1)^2 \qquad \mu_1 = 130$$

$$= \frac{1}{3}\left[(120-130)^2 + (140-130)^2 + \cdots\right]$$

$$= \frac{400}{3}$$

$$\sigma_2^2 = \frac{1}{M-1} \sum_{n=1}^{M} (x_{2,m} - \mu_2)^2 \qquad \mu_2 = 0.4$$

$$= \frac{1}{3}\left[(0.3-0.4)^2 + (0.5-0.4)^2 + \cdots\right]$$

$$= \frac{0.04}{3}$$

$$\Sigma_1 = \begin{bmatrix} 400/3 & 0 \\ 0 & 0.04/3 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} b_5 \\ v_5 \end{bmatrix}, \quad x_6 = \begin{bmatrix} b_6 \\ v_6 \end{bmatrix}$$

$$d_{\Sigma_1}^2(x_5, x_6) = (x_5 - x_6)^T \Sigma_1^{-1} (x_5 - x_6)$$

$$= \frac{(b_5 - b_6)^2}{400/3} + \frac{(v_5 - v_6)^2}{0.04/3}$$

$$d_{\Sigma_1}(x_5, x_6) = \sqrt{\frac{(b_5 - b_6)^2}{400/3} + \frac{(v_5 - v_6)^2}{0.04/3}}$$

$$d_{\Sigma_1}^2(x_5, x_6) = \begin{bmatrix} (b_5 - b_6) & (v_5 - v_6) \end{bmatrix} \begin{bmatrix} \frac{1}{400/3} & 0 \\ 0 & \frac{1}{0.04/3} \end{bmatrix} \begin{bmatrix} b_5 - b_6 \\ v_5 - v_6 \end{bmatrix}$$

17 $x_1, x_2, x_3 \; : \; y_1, y_2, y_3$

$$h_t(x_i) = \begin{cases} \text{CORRECT} & t \neq i \\ \text{INCORRECT} & t = i \end{cases}$$

$$w_{1,i} = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$$

$h_1(x_i)$: ERRORS: $[1, 0, 0]$

$$\epsilon_1 = \sum_i w_{1,i} \left| y_i - h_t(x_i) \right| = \frac{1}{3}$$

$$\beta_1 = \frac{\epsilon_1}{1-\epsilon_1} = \frac{1/3}{2/3} = \frac{1}{2} \qquad \alpha_1 = -\ln\beta_1 = \ln(2)$$

$$\tilde{w}_{2,i} = \left\{ \begin{array}{ll} w_{1,i} & \text{IF } h_1(x_i) \text{ INCORRECT} \\ \beta_1 w_{1,i} & \text{IF } h_1(x_i) \text{ CORRECT} \end{array} \right\} = \left[\frac{1}{3}, \frac{1}{6}, \frac{1}{6}\right]$$

$$w_{2,i} = \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right]$$

CHOOSE $h_2(x_1)$; ERROR ON TOKEN x_2

$$\epsilon_2 = \sum_i w_{2,i} \underline{\left| y_i - h_2(x_i) \right|} = \frac{1}{4}$$

$$\beta_2 = \frac{\epsilon_2}{1-\epsilon_2} = \frac{1/4}{3/4} = \frac{1}{3}, \qquad \alpha_2 = -\ln\beta_2 = \ln(3)$$

$$\tilde{w}_{3,i} = \left[\frac{1}{6}, \frac{1}{4}, \frac{1}{12}\right] \qquad \frac{1}{6} + \frac{1}{4} + \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$$

① $PR(A, B) = Pr(A|B)\,Pr(B)$

ⓒ $Pr(A) = \sum_{b \in B} Pr(A, B = b)$

$$w_{3,i} = \frac{\tilde{w}_{3,i}}{\sum_j \tilde{w}_{3,j}} = \left[\frac{\frac{1}{6}}{\frac{1}{2}}, \frac{\frac{1}{4}}{\frac{1}{2}}, \frac{\frac{1}{12}}{\frac{1}{2}}\right] = \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{6}\right]$$

CHOOSE h_3: CORRECT EXCEPT x_3

$$\epsilon_3 = \sum_i w_{3,i} \left| y_i - h_3(x_i) \right| = \frac{1}{6}$$

$$\beta_3 = \frac{\epsilon_3}{1-\epsilon_3} = \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5}, \quad \alpha_3 = -\ln(\beta_3) = \ln(5)$$

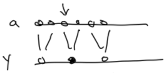
$$h(x) = \sum_t \alpha_t h_t(x)$$

$$= \ln(2)\, h_1(x) + \ln(3)\, h_2(x) + \ln(5)\, h_3(x)$$

$\boxed{21}$ $\quad a[n] = w\, x[n] \qquad 1 \le n \le 10$

$$y[k] = \max\left(0, \max_{2k-1 \le n \le 2k} a[n]\right)$$

$\epsilon[k] = \frac{\partial E}{\partial y[k]}$, FIND $\frac{\partial E}{\partial w}$ IN TERMS OF $x[n]$ AND $\epsilon[k]$



$$\frac{\partial E}{\partial a[n]} = \begin{cases} \frac{\partial E}{\partial y[k]} \\ 0 \end{cases}$$

$a[n] = \underset{2k-1 \le m \le 2k}{\arg\max}\, a[m]$ AND $a[n] \ge 0$

$$\frac{\partial E}{\partial w} = \underline{\underline{\sum_n \frac{\partial E}{\partial a[n]} \frac{\partial a[n]}{\partial w}}} = \sum_n \frac{\partial E}{\partial a[n]} x[n]$$

$$n^*(k) = \underset{n:\, 2k-1 \le m \le 2k}{\arg\max}\, a[m]$$

$$\frac{\partial E}{\partial a[n]} = \begin{cases} \frac{\partial E}{\partial \epsilon[k]} & \text{IF } n = n^*(k) \text{ AND } a[n] \ge 0 \\ 0 & \text{ELSE} \end{cases}$$

$$\frac{\partial E}{\partial w} = \sum_k \frac{\partial E}{\partial \epsilon[k]}\, x[n^*(k)]$$

$\boxed{22}$ GIVEN $\{(x_1, \zeta_1), \ldots, (x_n, \zeta_n)\}$

$$z_i = \sigma(w^T x_i), \quad \zeta_i \in \{0, 1\}$$

$$\mathcal{L} = -\frac{1}{n} \sum_{i=1}^{n} \zeta_i \ln z_i + (1-\zeta_i) \ln(1-z_i)$$
$$+ \frac{1}{2n} \sum_{i=1}^{n} (w^T x_i)^2$$

$$\frac{\partial \mathcal{L}}{\partial w} = -\frac{1}{n} \sum_{i=1}^{n} \frac{\zeta_i}{z_i} \frac{\partial z_i}{\partial w} + \frac{1-\zeta_i}{1-z_i} \frac{\partial (1-z_i)}{\partial w}$$
$$+ \frac{1}{n} \sum_{i=1}^{n} (w^T x_i)\, x_i^T$$

$$z_i = \sigma(w^T x_i)$$

$$\frac{\partial z_i}{\partial w} = \sigma'(w^T x_i) \frac{\partial (w^T x_i)}{\partial w} = \sigma'(w^T x_i)\, x_i^T$$
$$= \sigma(w^T x_i)(1 - \sigma(w^T x_i))\, x_i^T$$
$$= z_i (1 - z_i)\, x_i^T$$

$$\frac{d(1-z_i)}{dw} = -z_i(1-z_i)x_i^T$$

$$\frac{\partial \mathcal{L}}{\partial w} = -\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\cancel{z_i}}\cancel{z_i}(1-z_i)x_i^T - \frac{1-y_i}{\cancel{1-z_i}} z_i \cancel{(1-z_i)} x_i^T\right)$$

$$+\frac{1}{n}\sum_{i=1}^{n}(u^T x_i)x_i^T$$

$\boxed{24}$ GIVEN $$ii[n_1] = \sum_{m_1=0}^{n_1} i[m_1]$$

FIND $$c[n_1] = \sum_{m_1=n_1}^{N_1} i[m_1]$$

$$= \sum_{m_1=0}^{N_1} i[m_1] - \sum_{m_1=0}^{n_1-1} i[m_1]$$

$$= ii[N_1] - ii[n_1-1]$$

GIVEN $$ii = \sum_{m_1=0}^{n_1}\sum_{m_2=0}^{n_2} i[m_1, m_2]$$

FIND $$c = \sum_{m_1=n_1}^{N_1}\sum_{m_2=n_2}^{N_2} i[m_1, m_2]$$

$$= ii[N_1, N_2] - ii[N_1, n_2-1]$$

$$- ii[n_1-1, N_2] + ii[n_1-1, n_2-1]$$

$$c = \sum_{m_1=0}^{N_1}\sum_{m_2=0}^{N_2} i[m_1, m_2] - \sum_{m_1=0}^{N_1}\sum_{m_2=0}^{n_2-1} i[m_1, m_2]$$

$$- \sum_{m_1=0}^{n_1-1}\sum_{m_2=0}^{N_2} i[m_1, m_2] + \sum_{m_1=0}^{n_1-1}\sum_{m_2=0}^{n_2-1} i[m_1, m_2]$$