2021oct28

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$$p(X|\Lambda) = \sum_{j=1}^{4} \alpha_2(j)$$

$$\alpha_1(i) = \pi_i b_i(\vec{x}_1)$$

$$\alpha_2(j) = \sum_{i=1}^{4} \alpha_1(i)\, a_{ij}\, b_j(\vec{x}_2)$$

w_0

$$\alpha_1(i) = \begin{cases} 1 \cdot b_1(\vec{x}_1) & i=1 \\ 0 & \text{else} \end{cases}$$

$$= \begin{cases} N(\vec{x}_1 | \vec{\mu}_1 = \begin{bmatrix} 0\\0\\0 \end{bmatrix}, \Sigma_1 = I) & i=1 \\ 0 & \text{else} \end{cases}$$

$$\alpha_2(j) = \alpha_1(1)\, a_{1j}\, b_j(\vec{x}_2)$$

$$= N(\vec{x}_1 | \vec{\mu}_1, I)(0.25) N(\vec{x}_2 | \vec{\mu}_j, I)$$

$$p(X|w_0) = \sum_j \alpha_2(j)$$

$$= N(\vec{x}_1 | \mu_1, I) \cancel{\sum_j \frac{1}{4} N(\vec{x}_2 | \mu_j, I)}$$

w_1

$$\alpha_1(i) = \begin{cases} 1 \cdot b_4(\vec{x}_1) & i=4 \\ 0 & \text{else} \end{cases}$$

$$\alpha_1(i) = \begin{cases} N(\vec{x}_1 | \vec{\mu}_4 = \begin{bmatrix} -1\\-1\\1 \end{bmatrix}, \Sigma_1' = I) & i=4 \\ 0 & \text{else} \end{cases}$$

$$\alpha_2(j) = \alpha_1(4)\, a_{4j}\, b_j(\vec{x}_2)$$

$$= N(\vec{x}_1 | \vec{\mu}_4, I)(0.25) N(\vec{x}_2 | \vec{\mu}_j, I)$$

$$p(X|w_1) = \sum_j \alpha_2(j)$$

$$= N(\vec{x}_1 | \mu_4, I) \cancel{\sum_j \frac{1}{4} N(\vec{x}_2 | \mu_j, I)}$$

choose w_0 if

$$p(w_0)\, p(X|w_0) > p(w_1)\, p(X|w_1)$$

$$(0.7)\, p(X|w_0) > (0.3)\, p(X|w_1)$$

$$(0.7) N(\vec{x}_1 | \mu_1, I) > (0.3) N(\vec{x}_1 | \mu_4, I)$$

$$\ln(0.7) - \frac{1}{2}(\vec{x}_1 - \vec{\mu}_1)^T(\vec{x}_1 - \vec{\mu}_1) >$$
$$\ln(0.3) - \frac{1}{2}(\vec{x}_1 - \vec{\mu}_4)^T(\vec{x}_1 - \vec{\mu}_4)$$

$$\ln(0.7) \cancel{- \frac{1}{2}\|\vec{x}_1\|^2} + \cancel{\vec{\mu}_1^T \vec{x}_1 - \frac{1}{2}\|\vec{\mu}_1\|^2} >$$
$$\ln(0.3) \cancel{- \frac{1}{2}\|\vec{x}_1\|^2} + \vec{\mu}_4^T \vec{x}_1 - \frac{1}{2}\|\vec{\mu}_4\|^2$$

$$0 > \ln\left(\frac{0.3}{0.7}\right) + \vec{\mu}_4^T \vec{x}_1 - \frac{1}{2}\|\vec{\mu}_4\|^2$$

CHOOSE w_1 if

$$0 < \left(\ln\left(\frac{0.3}{0.7}\right) - \frac{3}{2}\right) + [-1, -1, 1]\, \vec{x}_1$$

$$\vec{w}_1 = \vec{\mu}_4 = \begin{bmatrix} -1\\-1\\1 \end{bmatrix}, \quad \vec{w}_2 = \begin{bmatrix} 0\\0\\0 \end{bmatrix}$$

$$b = \ln\left(\frac{0.3}{0.7}\right) - \frac{3}{2}$$