UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 1 Sample Problems

Problem 1.1

Consider the signal

$$x(t) = \sin(14000\pi t) + 3\cos(16000\pi t)$$

Suppose $x(t)$ is sampled at $F_s = 10,000$Hz without an anti-aliasing filter. Find $x[n]$—normalize the frequencies of both sinusoids so they're between 0 and π.

Problem 1.2

Same signal as in Problem 1. Find the CTFT, $X(j\Omega)$, and the DTFT, $X(e^{j\omega})$. Sketch them both, showing the frequencies and areas of the four impulses.

Problem 1.3

Consider the signal

$$x(t) = e^{-100\pi t}\sin(1000\pi t)\,u(t)$$

Find $X(j\Omega)$, using the formula $\int_0^\infty e^{at}dt = \frac{1}{a}$. Sketch $|X(j\Omega)|$, showing the approximate frequencies, heights, and widths of the two peaks.

Problem 1.4

Suppose, now, that the signal $x(t)$ from problem 3 is sampled at $F_s = 2000$ samples/second without an anti-aliasing filter, so that $x[n] = x(n/F_s)$. Find its DTFT using the formula $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$, which, as you know, is valid only if $|a| < 1$. Sketch $|X(e^{j\omega})|$, showing the approximate frequencies, heights, and widths of the two peaks. Note: in order to find the frequencies, heights, and widths of the two peaks, you may find it useful to approximate $e^a \approx 1 + a$.