UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Principles of Signal Analysis
Spring 2015

EXAM 2 SOLUTIONS

Thursday, April 9, 2015

Problem 1 (25 points)

The stock market alternates between long bull markets (state 1) and short bear markets (state 2). This HMM has the following parameters:

$$\vec{\pi} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.999 & 0.001 \\ 0.005 & 0.995 \end{bmatrix}, \quad \mu_1 = 0.1, \quad \mu_2 = -0.3, \quad \sigma_1^2 = \sigma_2^2 = 1,$$

where $\pi_i = p(q_1 = i)$, $a_{ij} = p(q_{t+1} = j | q_t = i)$, and $p(x_t | q_t = j) = \mathcal{N}(x_t; \mu_j, \sigma_j^2)$.

You observe x_2 on day 2.

For what values of x_2 does the forward algorithm yield probabilities $\alpha_t(i)$ such that $\alpha_2(2) > \alpha_2(1)$?

Asking exactly the same question in different words: for what values of x_2 would it be rational to conclude that a bear market has started?

SOLUTION: $x_2 < -0.1 - 2.5 \ln(999)$

Problem 2 (15 points)

A triangle begins at

$$X = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Notice that, in this triangle, the barycentric coordinates of any point (x_4, y_4) are given by

$$\vec{\lambda}_4 = \begin{bmatrix} 1 - x_4 - y_4 \\ x_4 \\ y_4 \end{bmatrix}$$

Suppose the triangle is rotated, shifted and scaled to the new position

$$\Xi = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 \\ \eta_1 & \eta_2 & \eta_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The point $(x_4, y_4) = \left(\frac{1}{3}, \frac{1}{3}\right)$, internal to the first triangle, gets mapped to some point (ξ_4, η_4). Find ξ_4 and η_4.

SOLUTION: $\xi_4 = \frac{1}{3}, \quad \eta_4 = 2$

Problem 3 (10 points)

Suppose a particular image has the following pixel values:

$$a[0,0] = 1, \quad a[1,0] = 0, \quad a[0,1] = 0, \quad a[1,1] = 0$$

Use bilinear interpolation to estimate the value of the pixel $a\left(\frac{1}{3}, \frac{1}{3}\right)$.

SOLUTION: $a\left(\frac{1}{3}, \frac{1}{3}\right) = \frac{4}{9}$

Problem 4 (25 points)

Consider two PDFs. Class $y = 0$ is Gaussian:

$$p(x|y=0) = \mathcal{N}\left(x; \mu_0, \sigma_0^2\right)$$

Class $y = 1$ is mixture Gaussian, and for some reason, one of its mixture components is the Gaussian from class 0:

$$p(x|y=1) = 0.9p(x|y=0) + 0.1\mathcal{N}\left(x; \mu_1, \sigma_1^2\right)$$

where $\mu_0 = 0$, $\mu_1 = 3$, and $\sigma_0^2 = \sigma_1^2 = 1$.

For what values of x is

$$\frac{p(x|y=1)}{p(x|y=0)} > 1?$$

SOLUTION: $x > \frac{3}{2}$

Problem 5 (25 points)

You have six audio training examples:

$$\vec{a}_1 = \begin{bmatrix} 400 \\ 0 \end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \vec{a}_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \vec{a}_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \vec{a}_5 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad \vec{a}_6 = \begin{bmatrix} -400 \\ 0 \end{bmatrix}$$

You also have six video training examples:

$$v_1 = -5, \quad v_2 = -3, \quad v_3 = -1, \quad v_4 = 1, \quad v_5 = 3, \quad v_6 = 5$$

The class labels for these audiovisual training data are as follows:

$$y_1 = 0, \quad y_2 = 0, \quad y_3 = 0, \quad y_4 = 1, \quad y_5 = 1, \quad y_6 = 1$$

Define the classification function $f(\vec{a}, v, y) = \lambda \ln p(y|\vec{a}) + (1-\lambda) \ln p(y|v)$, where

- $p(y|\vec{a})$ is estimated using a **3-nearest neighbor (3NN)** pmf,
- $p(y|v)$ is estimated using a **5-nearest neighbor (5NN)** pmf.

Suppose $v_7 = 2$, and $\vec{a}_7 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$. For what values of λ is $f(\vec{a}_7, v_7, 1) > f(\vec{a}_7, v_7, 0)$?

SOLUTION: $\lambda < \frac{\ln(3/2)}{\ln(3)}$