UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 MULTIMEDIA SIGNAL PROCESSING
Fall 2019

EXAM 1 SOLUTIONS

Tuesday, September 24, 2019

Problem 1   (20 points)

(a) $T = 98$

(b)
$$N = 480 \left(\frac{18000}{5000} \right) = 1728$$

(c)
$$a_{kn} = e^{-j\frac{2\pi kn}{N}}$$

(d)
$$S[k,t] = \max \left(0, \left(\frac{255}{60} \right) 20 \log_{10} \left(\frac{|X[k,t]|}{X_{MAX}/1000} \right) \right)$$

Problem 2   (5 points)

$$X[k] = \frac{1}{2} W_R \left(\frac{2\pi k}{N} - \omega_0 \right) + \frac{1}{2} W_R \left(\frac{2\pi k}{N} + \omega_0 \right)$$

Problem 3   (5 points)

$$S_{xx}(\omega) = \frac{1}{1 - \rho e^{-j\omega}} + \frac{1}{1 - \rho e^{j\omega}} - 1$$

Problem 4   (5 points)

Any solution that is even-symmetric ($s[m] = s[M - m]$), and that includes all and only the samples of $x[n]$, is acceptable. For example, here are two possibilities:

$$M = 2N \text{ and } s[m] = \begin{cases} x[m] & m \in \{0, 1, \ldots, N - 1\} \\ 0 & m = N \\ x[M - m] & m \in \{N + 1, \ldots, M - 1\} \end{cases}$$

or

$$M = 2N \text{ and } s[m] = \begin{cases} x\left[m - \frac{1}{2}\right] & m \in \left\{\frac{1}{2}, \frac{3}{2}, \ldots, N - \frac{1}{2}\right\} \\ x\left[M - m - \frac{1}{2}\right] & m \in \left\{N + \frac{1}{2}, \ldots, M - \frac{1}{2}\right\} \end{cases}$$

Problem 5   (15 points)

(a)

$$Y^T Y = \begin{bmatrix} \lambda_0 & 0 & \ldots & 0 \\ 0 & \lambda_1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \lambda_{D-1} \end{bmatrix}$$

(b)

$$\sum_{m=0}^{M-1} \|\vec{x}_m\|_2^2 = \sum_{d=0}^{D-1} \lambda_d$$

(c)

$$\vec{v}_i^T X^T X \vec{v}_j = \begin{cases} \lambda_i & i = j \\ 0 & \text{otherwise} \end{cases}$$