UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Multimedia Signal Processing
Fall 2019

PRACTICE EXAM 2

Tuesday, October 22, 2019

- This is a PRACTICE exam. On the real exam, you'll be allowed to use one sheet (front and back) of handwritten notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- The real exam will have a total of 50 points. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Possibly Useful Formulas

YPbPr and Sobel Mask

$$\begin{bmatrix} Y \\ P_b \\ P_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.168736 & -0.331264 & 0.5 \\ 0.5 & -0.418688 & -0.081312 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$G_x[n_1,n_2] = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} ** I[n_1,n_2], \quad G_y[n_1,n_2] = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} ** I[n_1,n_2]$$

Integral Image and Lowpass Filter

$$ii[n_1,n_2] = \sum_{m_1=0}^{n_1} \sum_{m_2=0}^{n_2} i[m_1,m_2]$$

$$H(\omega_1,\omega_2) = \begin{cases} 1 & |\omega_1| < \phi_1, \quad |\omega_2| < \phi_2 \\ 0 & \text{otherwise} \end{cases} \qquad h[n_1,n_2] = \left(\frac{\phi_1}{\pi}\right)\left(\frac{\phi_2}{\pi}\right) \operatorname{sinc}(\phi_1 n_1) \operatorname{sinc}(\phi_2 n_2)$$

Orthogonality Principle and LPC

$$\varepsilon = E\left[\left(x[n] - \sum_{m=1}^{p} \alpha_m x[n-m]\right)^2\right], \quad \frac{\partial \varepsilon}{\partial \alpha_k} = -2E\left[x[n-k]\left(x[n] - \sum_{m=1}^{12} \alpha_m x[n-m]\right)\right]$$

$$R_{xx}[k] = \sum_{m=1}^{12} \alpha_m R_{xx}[k-m]$$

Autocorrelation and Power Spectrum

$$R_{xx}[n] = E\{x[m]x[m-n]\} \leftrightarrow S_{xx}(\omega) = \sum_{n=-\infty}^{\infty} R_{xx}[n] e^{-j\omega n}$$

$$r_{xx}[n] = \sum_{m=-\infty}^{\infty} x[m]x[m-n] \leftrightarrow s_{xx}(\omega) = \sum_{n=-\infty}^{\infty} r_{xx}[n] e^{-j\omega n}$$

Fourier Series

$$x[n] = \sum_{k=0}^{P-1} X_k e^{j2\pi kn/P} \qquad X_k = \frac{1}{P} \sum_{n=0}^{P-1} x[n] e^{-j2\pi kn/P}$$

Autocorrelation and Power Spectrum

$$R_{xx}[n] = E\{x[m]x[m-n]\} \leftrightarrow S_{xx}(\omega) = \sum_{n=-\infty}^{\infty} R_{xx}[n] e^{-j\omega n}$$

$$r_{xx}[n] = \sum_{m=-\infty}^{\infty} x[m]x[m-n] \leftrightarrow s_{xx}(\omega) = \sum_{n=-\infty}^{\infty} r_{xx}[n] e^{-j\omega n}$$

Problem 1 (30 points)

Suppose you have an RGB image $i[n_1, n_2, n_3]$ with $0 \leq n_1 < N_1$ rows, $0 \leq n_2 < N_2$ columns, and $0 \leq n_3 < 3$ color planes. The matched filter in part (d) of this problem is of size $M_1 \times M_2$. Use big-$\mathcal{O}$ notation, in terms of the variables N_1, N_2, M_1 and M_2, to express the complexity of each of the following operations:

(a) Coverting from RGB to YPbPr color space.

(b) Computing the horizontal and vertical gradients of each color plane using a Sobel mask.

(c) Lowpass filtering (after zero-padding, so that the output is of the same size, $N_1 \times N_2 \times 3$, as the input) with a separable ideal anti-aliasing filter whose frequency response is

$$H(\omega_1, \omega_2) = \begin{cases} 1 & |\omega_1| < \frac{\pi}{3}, \quad |\omega_2| < \frac{\pi}{3} \\ 0 & \text{otherwise} \end{cases}$$

(d) Filtering with a matched filter of size M_1 rows, M_2 columns.

(e) Calculating the integral image $ii[n_1, n_2] = \sum_{m_1=0}^{n_1} \sum_{m_2=0}^{n_2} i[m_1, m_2, 0]$ for all $0 \leq n_1 < N_1$ and $0 \leq n_2 < N_2$.

(f) Given the integral image, find the box-summation $f[b_1, b_2, e_1, e_2]$ defined as

$$f[b_1, b_2, e_1, e_2] = \sum_{n_1=b_1}^{e_1} \sum_{n_2=b_2}^{e_2} i[n_1, n_2, 0]$$

for all values $0 \leq b_1 \leq N_1 - 1$, $0 \leq e_1 \leq N_1 - 1$, $0 \leq b_2 \leq N_2 - 1$, $0 \leq e_2 \leq N_2 - 1$.

Problem 2 (10 points)

Suppose you have an input image with 8-bit integer pixel values, $0 \leq i[n_1, n_2, n_3] \leq 255$, where n_1 is the row index, n_2 is the column index, and n_3 is the color plane. What are the minimum and maximum pixel values that result as the outputs of the following operations:

(a) Convert to a YPbPr color space. What are the minimum and maximum possible values of Y, P_b, and P_r?

(b) Compute the horizontal and vertical gradients using a Sobel mask. What are the minimum and maximum possible values of each of the two gradient images?

Problem 3 (5 points)

Consider the infinite-sized image $i[n_1, n_2] = \delta[n_1 - 5]$, i.e.,

$$i[n_1, n_2] = \begin{cases} 1 & n_1 = 5 \\ 0 & \text{otherwise} \end{cases}$$

Use a Sobel mask to find the resulting images $G_x[n_1, n_2]$ and $G_y[n_1, n_2]$.

Problem 4 (5 points)

Suppose you want to find the horizon line in a grayscale image $i[n_1,n_2]$. Suppose the horizon line is defined to be the row index n_1 that maximizes the brightness difference $BD[n_1]$, defined as

$$BD[n_1] = \sum_{m_2=0}^{N_2-1}\left(\left(\frac{1}{n_1}\sum_{m_1=0}^{n_1-1} i[m_1,m_2]\right) - \left(\frac{1}{N_1-n_1}\sum_{m_1=n_1}^{N_1-1} i[m_1,m_2]\right)\right)$$

You are given the integral image $ii[n_1,n_2] = \sum_{m_1=0}^{n_1}\sum_{m_2=0}^{n_2} i[m_1,m_2,0]$. Devise a formula that uses $ii[n_1,n_2]$ to compute $BD[n_1]$ with a small constant number of operations per candidate horizon line.

Problem 5 (5 points)

Consider the problem of upsampling, by a factor of 2, the infinite-sized image

$$x[n_1,n_2] = \delta[n_1-5] = \begin{cases} 1 & n_1 = 5 \\ 0 & \text{otherwise} \end{cases}$$

Suppose that the image is upsampled, then filtered, as

$$y[n_1,n_2] = \begin{cases} x[n_1/2,n_2/2] & n_1/2 \text{ and } n_2/2 \text{ both integers} \\ 0 & \text{otherwise} \end{cases} \qquad z[n_1,n_2] = y[n_1,n_2] ** h[n_1,n_2]$$

Let $h[n_1,n_2]$ be the ideal anti-aliasing filter with frequency response

$$H(\omega_1,\omega_2) = \begin{cases} 1 & |\omega_1| < \frac{\pi}{2},\quad |\omega_2| < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

Find $z[n_1,n_2]$.

Problem 6 (5 points)

The stochastic autocorrelation of a periodic signal is periodic, $R_{xx}[P] = R_{xx}[0]$. How about the signal autocorrelation? Suppose that the frame length is an integer multiple of the number of periods, $L = kP$, so that

$$x[n] = \begin{cases} \text{periodic with period } P & 0 \le n \le kP-1 \\ 0 & \text{otherwise} \end{cases}$$

Find $r_{xx}[P]$ in terms of $r_{xx}[0]$.

Problem 7 (10 points)

Consider the signal $x[n] = \beta^n u[n]$, where $u[n]$ is the unit step function.

(a) Find the LPC coefficient, α, that minimizes ε, where

$$\varepsilon = \sum_{n=-\infty}^{\infty} e^2[n], \quad e[n] = x[n] - \alpha x[n-1]$$

(b) Find the signal $e[n]$ that results from your choice of α in part (a).

Problem 8 (10 points)

Consider the LPC synthesis filter $s[n] = e[n] + \alpha s[n-1]$.

(a) Under what condition on α is the synthesis filter stable?

(b) Assume that the synthesis filter is stable. Suppose that $e[n]$ is the pulse train $e[n] = \sum_{p=-\infty}^{\infty} \delta[n - pP]$. As a function of α, P, and ω, what is the DTFT $S(e^{j\omega})$? You need not simplify, but your answer should contain no integrals or infinite sums.