UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Multimedia Signal Processing
Fall 2019

EXAM 2

Tuesday, October 22, 2019

- This is a CLOSED BOOK exam. You may use one sheet (front and back) of handwritten notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- There are a total of 50 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

Name: ____

netid: ____

Possibly Useful Formulas

YPbPr and Sobel Mask

$$\begin{bmatrix} Y \\ P_b \\ P_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.168736 & -0.331264 & 0.5 \\ 0.5 & -0.418688 & -0.081312 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$G_x[n_1,n_2] = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} * * I[n_1,n_2], \quad G_y[n_1,n_2] = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} * * I[n_1,n_2]$$

Integral Image and Lowpass Filter

$$ii[n_1,n_2] = \sum_{m_1=0}^{n_1} \sum_{m_2=0}^{n_2} i[m_1,m_2]$$

$$H(\omega_1,\omega_2) = \begin{cases} 1 & |\omega_1| < \phi_1, \quad |\omega_2| < \phi_2 \\ 0 & \text{otherwise} \end{cases} \qquad h[n_1,n_2] = \left(\frac{\phi_1}{\pi}\right)\left(\frac{\phi_2}{\pi}\right) \operatorname{sinc}(\phi_1 n_1) \operatorname{sinc}(\phi_2 n_2)$$

Orthogonality Principle and LPC

$$\varepsilon = E\left[\left(x[n] - \sum_{m=1}^{p} \alpha_m x[n-m]\right)^2\right], \quad \frac{\partial \varepsilon}{\partial \alpha_k} = -2E\left[x[n-k]\left(x[n] - \sum_{m=1}^{12} \alpha_m x[n-m]\right)\right]$$

$$R_{xx}[k] = \sum_{m=1}^{12} \alpha_m R_{xx}[k-m]$$

Fourier Series

$$x[n] = \sum_{k=0}^{P-1} X_k e^{j2\pi kn/P} \quad X_k = \frac{1}{P}\sum_{n=0}^{P-1} x[n] e^{-j2\pi kn/P}$$

Autocorrelation and Power Spectrum

$$R_{xx}[n] = E\left\{x[m]x[m-n]\right\} \leftrightarrow S_{xx}(\omega) = \sum_{n=-\infty}^{\infty} R_{xx}[n] e^{-j\omega n}$$

$$r_{xx}[n] = \sum_{m=-\infty}^{\infty} x[m]x[m-n] \leftrightarrow s_{xx}(\omega) = \sum_{n=-\infty}^{\infty} r_{xx}[n] e^{-j\omega n}$$

Problem 1 (10 points)

Consider an unvoiced zero-mean Gaussian random signal, $x[n] \sim \mathcal{N}(0,1)$, with the following autocorrelation:

$$R_{xx}[n] = e^{-\beta|n|}$$

Suppose $e[n] = x[n] - \alpha x[n-1]$.

(a) Find α to minimize $E\left[e^2[n]\right]$.

(b) Find the value of $E\left[e^2[n]\right]$ that results from the α you chose in part (a).

Problem 2 (5 points)

Consider the synthesis filter $s[n] = e[n] + bs[n-1] - \left(\frac{b}{2}\right)^2 s[n-2]$. For what values of b is the synthesis filter stable?

Problem 3 (5 points)

Suppose $e[n] \sim \mathcal{N}(0,1)$ is Gaussian white noise, $R_{ee}[n] = \delta[n]$. Consider the synthesis filter, $s[n] = e[n] + \alpha s[n-1]$. Find the power spectrum of the synthesized signal, $S_{ss}(\omega)$, in terms of ω and α. You need not simplify, but your answer should contain no integrals or infinite sums.

Problem 4 (5 points)

You are given the integral image $ii[n_1, n_2]$, defined in terms of the image $i[n_1, n_2]$ as

$$ii[n_1, n_2] = \sum_{m_1=0}^{n_1} \sum_{m_2=0}^{n_2} i[m_1, m_2]$$

Write an equation that computes the complementary integral image, $c[n_1, n_2]$, in a small constant number of operations per output pixel, where

$$c[n_1, n_2] = \sum_{m_1=n_1}^{N_1-1} \sum_{m_2=n_2}^{N_2-1} i[m_1, m_2]$$

Problem 5 (5 points)

Suppose you have a 200×200-pixel image that is just one white dot at pixel $(45, 25)$, and all the other pixels are black:

$$x[n_1, n_2] = \begin{cases} 255 & n_1 = 45,\ n_2 = 25 \\ 0 & \text{otherwise},\ 0 \le n_1 < 199,\ 0 \le n_2 < 199 \end{cases}$$

This image is upsampled to size 400×400, then filtered, as

$$y[n_1, n_2] = \begin{cases} x[n_1/2, n_2/2] & n_1/2 \text{ and } n_2/2 \text{ both integers} \\ 0 & \text{otherwise} \end{cases} \qquad z[n_1, n_2] = y[n_1, n_2] * *h[n_1, n_2]$$

where $h[n_1, n_2]$ is the ideal anti-aliasing filter whose frequency response is

$$H(\omega_1, \omega_2) = \begin{cases} 1 & |\omega_1| < \frac{\pi}{2},\ |\omega_2| < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

Find $z[n_1, n_2]$.

Problem 6 (10 points)

Consider an infinite-sized RGB image containing a single diagonal white line on a black background, specifically

$$R[n_1, n_2] = G[n_1, n_2] = B[n_1, n_2] = \begin{cases} 255 & n_1 - n_2 = 5 \\ 0 & \text{otherwise} \end{cases}$$

where $-\infty < n_1 < \infty$, $-\infty < n_2 < \infty$, and the signals R, G, and B are the red, green, and blue channels, respectively.

(a) Find the luminance $Y[n_1, n_2]$.

(b) Find the blue-shift $P_b[n_1, n_2]$.

Problem 7 (10 points)

Consider an infinite-sized grayscale image of a diagonal gray line:

$$x[n_1,n_2] = \begin{cases} 105 & n_1 - n_2 = 5 \\ 0 & \text{otherwise} \end{cases}, \quad -\infty < n_1 < \infty, \quad -\infty < n_2 < \infty$$

NOTE: EACH part of this problem is a **1D ROW CONVOLUTION** of the ORIGINAL IMAGE with a **DIFFERENT** row filter. Although these filters are related to the Sobel mask, NEITHER PART OF THIS PROBLEM IMPLEMENTS A COMPLETE SOBEL MASK.

(a) Suppose we **convolve each row** with a differencing filter:

$$y[n_1,n_2] = x[n_1,n_2] * d_2[n_2], \quad d_2[n_2] = \begin{cases} 1 & n_2 = 0 \\ -1 & n_2 = 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $y[n_1,n_2]$.

(b) Suppose, INSTEAD, that we **convolve each row** with an averaging filter

$$z[n_1,n_2] = x[n_1,n_2] * a_2[n_2], \quad a_2[n_2] = \begin{cases} 1 & n_2 \in \{0,2\} \\ 2 & n_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $z[n_1,n_2]$.