Problem 1 (10 points)

(a) $D(\vec{x},\vec{y})$ is not positive definite, e.g., consider $\vec{x}=[1,0,0]^T$, $\vec{y}=[0,1,0]^T$.

(b) Consider the numbers

$$\begin{aligned} a_1 &= (x_1-y_1)^2-(x_1-z_1)^2 \\ a_2 &= (x_2-y_2)^2-(x_2-z_2)^2 \\ a_3 &= (x_3-y_3)^2-(x_3-z_3)^2 \end{aligned}$$

It is necessary that

$$\frac{a_1}{\sigma_1^2}+\frac{a_2}{\sigma_2^2}+\frac{a_3}{\sigma_3^2}>0$$

If at least one of the a_k is positive, then we can choose

$$\sigma_k^2=\begin{cases} 100a_k & a_k>0 \\ 1 & a_k=0 \\ -0.01a_k & a_k<0 \end{cases}$$

If all of the a_k are negative, then there's no positive-definite Σ that will solve the problem. The problem statement didn't specify that Σ has to be positive-definite, though, so we could just choose

$$\sigma_k^2=\mathrm{sign}(a_k)$$

Problem 2 (10 points)

(a) The sketch should show the line $x_1=0$.

(b) $1-\Phi(2)$

Problem 3 (10 points)

(a) The sketch should show an ellipse centered at the origin, with its axes at 45 degrees relative to the main axes, passing through the points $(\sqrt{2},\sqrt{2})$, $(-\sqrt{2},-\sqrt{2})$, $(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2})$, and $(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$.

(b) $[4\sqrt{2},0]^T$.

Problem 4 (10 points)

(a) The sketch should show an empty plus-sign, the figure $\min(|x_1|,|x_2|)=0.5$.

(b) The solution can be any value of η in the range $\frac{1}{2}e^{1/2}>\eta>\frac{1}{2}e^{1/4}$.