UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Multimedia Signal Processing
Fall 2017

EXAM 3

Friday, December 15, 2017

- This is a CLOSED BOOK exam. You may use one sheet (front and back) of handwritten notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- There are a total of 40 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
Total	

Name: ____

Possibly Useful Formulas

Scaled Forward-Backward Algorithm

$$
\begin{aligned}
\tilde{\alpha}_1(i) &= \pi_i b_i(\vec{x}_1) \\
g_t &= \sum_{i=1}^{N} \tilde{\alpha}_t(i) \\
\hat{\alpha}_t(i) &= \frac{1}{g_t}\tilde{\alpha}_t(i) \\
\tilde{\alpha}_t(i) &= \sum_{j=1}^{N} \hat{\alpha}_{t-1}(j) a_{ji} b_i(\vec{x}_t) \\
\hat{\beta}_T(i) &= 1 \\
\tilde{\beta}_t(i) &= \sum_{j=1}^{N} \hat{\beta}_{t+1}(j) a_{ij} b_j(\vec{x}_{t+1}) \\
\hat{\beta}_t(i) &= \frac{1}{g_{t+1}}\tilde{\beta}_t(i)
\end{aligned}
$$

Adaboost Assume $y_i, h_j(x_i) \in \{0, 1\}$. For $t = 1, \ldots, T$:

$$
\begin{aligned}
h_t^* &= \arg\min_j \sum_{i=1}^{n} w_{t,i} \left| h_j(x_i) - y_i \right| \\
\epsilon_t &= \sum_{i=1}^{n} w_{t,i} \left| h_t^*(x_i) - y_i \right| \\
\tilde{w}_{t+1,i} &= \begin{cases} \frac{\epsilon_t}{1-\epsilon_t} w_{t,i} & h_t^*(x_i) = y_i \\ w_{t,i} & \text{otherwise} \end{cases} \\
w_{t+i,i} &= \frac{\tilde{w}_{t+1,i}}{\sum_j \tilde{w}_{t+1,j}} \\
H(x) &= u\left(\sum_{t=1}^{T} \alpha_t (h_t^*(x) - \frac{1}{2}) \right) \\
\alpha_t &= \log \frac{1-\epsilon_t}{\epsilon_t}
\end{aligned}
$$

Affine Transforms and Barycentric Coordinates

$$
\begin{aligned}
\vec{x}_0 &= [\vec{x}_1, \vec{x}_2, \vec{x}_3]\vec{\lambda} \\
\vec{u}_i &= \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \vec{x}_i \\
\vec{u}_0 &= [\vec{u}_1, \vec{u}_2, \vec{u}_3]\vec{\lambda}
\end{aligned}
$$

Problem 1 (10 points)

A particular two-layer neural network accepts a two-dimensional input vector $\vec{x} = [x_1, x_2, 1]^T$, and generates an output $z = h(\vec{v}^T g(U\vec{x}))$. Choose network weights $\vec{v}$ and U, and element-wise scalar nonlinearities $h()$ and $g()$, that will generate the following output:

$$z = \begin{cases} 1 & |x_1| < 2 \text{ and } |x_2| < 2 \\ -1 & \text{otherwise} \end{cases}$$

Problem 2 (10 points)

A reference image $I_0(u,v)$ has the following pixel values:

$$I_0(u,v) = 1 + (-1)^{u+v}$$

The test image $I_1(x,y)$ is created by piece-wise affine transformation of the pixel locations in $I_0(u,v)$. In particular, the triangle U in $I_0(u,v)$ is moved to the triangle X in $I_1(x,y)$, where

$$U = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) Find the reference coordinate $\vec{u} = [u,v,1]^T$ that corresponds to the test coordinate $\vec{x} = [3,2,1]^T$.

(b) Use bilinear interpolation to find the value of the test pixel $I_1(3,2)$.

Problem 3 (10 points)

A bimodal HMM uses a common state sequence, $Q = [q_1, \ldots, q_T]$, to explain two different observation sequences $X = [\vec{x}_1, \ldots, \vec{x}_T]$ and $Y = [\vec{y}_1, \ldots, \vec{y}_T]$. The HMM is parameterized by

$$\begin{aligned} \pi_i &= p(q_1 = i) \\ a_{ij} &= p(q_t = j | q_{t-1} = i) \\ b_j(\vec{x}_t) &= p_X(\vec{x}_t | q_t = j) \\ c_j(\vec{y}_t) &= p_Y(\vec{y}_t | q_t = j) \end{aligned}$$

Define

$$\begin{aligned} \alpha_t(i) &= p(\vec{x}_1, \vec{y}_1, \ldots, \vec{x}_t, \vec{y}_t, q_t = i) \\ \beta_t(i) &= p(\vec{x}_{t+1}, \vec{y}_{t+1}, \ldots, \vec{x}_T, \vec{y}_T | q_t = i) \end{aligned}$$

(a) Specify initialization formulas for $\alpha_1(i)$ and $\beta_T(i)$ in terms of π_i, a_{ij}, $b_j(\vec{x}_t)$, and $c_j(\vec{x}_t)$.

(b) Specify iteration formulas for $\alpha_t(i)$ and $\beta_t(i)$ in terms of π_i, a_{ij}, $b_j(\vec{x}_t)$, $c_j(\vec{x}_t)$, $\alpha_{t-1}(j)$, and $\beta_{t+1}(j)$.

Problem 4 (10 points)

Suppose that you have a training database with three training vectors $\vec{x}_1$, $\vec{x}_2$, and $\vec{x}_3$ whose correct labels are y_1, y_2, and y_3. You also have a set of three weak classifiers h_1, h_2, and h_3, each of which is right for exactly two of the three training tokens, as follows:

$$h_t(\vec{x}_i) = \begin{cases} y_i & i \neq t \\ \text{incorrect} & i = t \end{cases}$$

Adaboost begins with the weights $w_{1,i} = \frac{1}{3}$, and runs for three iterations, resulting in the strong classifier

$$H(\vec{x}) = \sum_{t=1}^{3} \alpha_t h_t(\vec{x})$$

You may assume that the weak classifiers are selected in order: h_1 is selected in the first iteration of Adaboost, h_2 in the second iteration, and h_3 in the third iteration. Find α_1, α_2, and α_3.