

Lecture 25: Exam 2 Review

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ECE 401: Signal and Image Analysis

- 1 Exam administration
- 2 Topics included in the exam
- 3 Sample problems

Outline

- 1 Exam administration
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When, Where

- When: Monday, November 2, 1:00-1:50pm.
- Where: ECEB 3015.
- Online section: Be signed on to zoom 12:55pm, with your camera on. Have Gradescope open on your phone or device, so you can upload your answers promptly at 1:50pm when the exam is completed.

What to bring

- Bring: Pencil or pen. Erasers.
- Notes sheet: You'll be given one formula sheet. You are also permitted to bring your own formula sheet. Yours should be 8.5x11, handwritten, both sides.
- Don't bring: Anything electronic. Any of your own paper (the last page of the exam paper will be blank, so you can use it as scratch paper).

Grading

- Exam will be graded on Gradescope.
- There will probably be four problems, each worth about 25 points.
- Grades should be available one week after the exam.

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Topics included in the exam: Lectures 15-22

- 1 DTFT, Windowing, and Windows
- 2 Filter design: Ideal filters, Windowing
- 3 DTFT of tones: Delta function, Fourier series, Spectral estimation
- 4 Overlap-Add

DTFT

The DTFT (discrete time Fourier transform) of any signal is $X(\omega)$, given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$
$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$$

Particular useful examples include:

$$f[n] = \delta[n] \leftrightarrow F(\omega) = 1$$
$$g[n] = \delta[n - n_0] \leftrightarrow G(\omega) = e^{-j\omega n_0}$$

Properties of the DTFT

Properties worth knowing include:

- ① Periodicity: $X(\omega + 2\pi) = X(\omega)$
- ① Linearity:

$$z[n] = ax[n] + by[n] \leftrightarrow Z(\omega) = aX(\omega) + bY(\omega)$$

- ② Time Shift: $x[n - n_0] \leftrightarrow e^{-j\omega n_0} X(\omega)$
- ③ Frequency Shift: $e^{j\omega_0 n} x[n] \leftrightarrow X(\omega - \omega_0)$
- ④ Filtering and Windowing:

$$y[n] = h[n] * x[n] \leftrightarrow Y(\omega) = H(\omega)X(\omega)$$

$$y[n] = w[n]x[n] \leftrightarrow Y(\omega) = \frac{1}{2\pi} W(\omega) * X(\omega)$$

Ideal Filters

- Ideal Lowpass Filter:

$$H_{LP}(\omega) = \begin{cases} 1 & |\omega| \leq \omega_c, \\ 0 & \omega_c < |\omega| \leq \pi. \end{cases} \quad \leftrightarrow \quad h_{LP}[m] = \frac{\omega_c}{\pi} \text{sinc}(\omega_c n)$$

- Ideal Highpass Filter:

$$H_{HP}(\omega) = 1 - H_{LP}(\omega) \quad \leftrightarrow \quad h_{HP}[n] = \delta[n] - \frac{\omega_c}{\pi} \text{sinc}(\omega_c n)$$

- Ideal Bandpass Filter:

$$H_{BP}(\omega) = H_{LP, \omega_2}(\omega) - H_{LP, \omega_1}(\omega) \\ \leftrightarrow h_{BP}[n] = \frac{\omega_2}{\pi} \text{sinc}(\omega_2 n) - \frac{\omega_1}{\pi} \text{sinc}(\omega_1 n)$$

Windows

- Rectangular Window:

$$w_R[n] = \begin{cases} 1 & -(\frac{L-1}{2}) \leq n \leq (\frac{L-1}{2}) \\ 0 & \text{otherwise} \end{cases} \leftrightarrow W_R(\omega) = \frac{\sin(\omega L/2)}{\sin(\omega/2)}$$

- Hamming and Hann Windows:

$$w_H[n] = w_R[n] \left(A + (1 - A) \cos \left(\frac{2\pi n}{L-1} \right) \right)$$

$$\begin{aligned} \leftrightarrow W_H(\omega) &= A W_R(\omega) + \frac{1-A}{2} W_R \left(\omega - \frac{2\pi}{L-1} \right) \\ &\quad + \frac{1-A}{2} W_R \left(\omega + \frac{2\pi}{L-1} \right) \end{aligned}$$

Pure Tones

- DTFT of a complex exponential is a delta function:

$$e^{j\omega_0 n} \leftrightarrow 2\pi\delta(\omega - \omega_0)$$

- DTFT of a cosine is two delta functions:

$$\cos(\omega_0 n) \leftrightarrow \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

- DTFT of a windowed cosine is frequency-shifted window functions:

$$\cos(\omega_0 n)w[n] \leftrightarrow \frac{1}{2}W(\omega - \omega_0) + \frac{1}{2}W(\omega + \omega_0)$$

Complex Tones

If $x[n]$ is periodic with Fourier series coefficients, then its DTFT and windowed DTFT are

$$\mathcal{F}\{x[n]\} = 2\pi \sum_{k=-N_0/2}^{(N_0-1)/2} X_k \delta\left(\omega - \frac{2k\pi}{N_0}\right)$$

$$\mathcal{F}\{w[n]x[n]\} = \sum_{k=-N_0/2}^{(N_0-1)/2} X_k W\left(\omega - \frac{2k\pi}{N_0}\right)$$

The Overlap-Add Algorithm

- 1 Divide $x[n]$ into frames of length M , zero-pad to $N \geq L + M - 1$, then FFT:

$$x_t[n] = x[n + tM]w[n]$$

$$X_t[k] = \text{FFT}\{x_t[n]\}$$

- 2 Generate the output from each frame

$$Y_t[k] = X_t[k]H[k]$$

$$y_t[n] = \text{FFT}^{-1}\{y_t[k]\}$$

- 3 Overlap the outputs, and add them together

$$y[n] = \sum_t y_t[n - tM]$$

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Sample problems

If there's time, at this point in the lecture, I will do some sample problems from the practice exam. If there are problems whose solution you'd like to see, please ask.