

2024 Sep 30

$$y[n] = 3x[n-15] + 3x[n-21] = \sum_m h[m]x[n-m]$$

What is $|H(\omega)|$ and $\angle H(\omega)$?

$$h[m] = \begin{cases} 3 & m=15 \\ 3 & m=21 \\ 0 & \text{else} \end{cases} \quad h[m] = 3\delta[m-15] + 3\delta[m-21]$$

$$\begin{aligned} H(\omega) &= \sum_m h[m] e^{-j\omega m} \\ &= 3e^{-j\omega 15} + 3e^{-j\omega 21} \\ &= 3e^{-j\omega 15} (1 + e^{-j\omega 6}) \\ &= 3e^{-j\omega 18} (e^{j\omega 3} + e^{-j\omega 3}) \\ &= 6e^{-j\omega 18} \cos(3\omega) \end{aligned}$$

$$|H(\omega)| = 6 |\cos(3\omega)|$$

$$\angle H(\omega) = \begin{cases} -18\omega & \cos(3\omega) > 0 \\ -18\omega + \angle(-1) \\ = -18\omega + \pi & \cos(3\omega) < 0 \end{cases}$$

Last line: Because $e^{j\pi} = -1$