2024 October 30

$$w[n] = \begin{cases} 1 & 0 \le n < 52 \\ 0 & \text{otherwise} \end{cases}$$

$$W[k] = W(\omega)\Big|_{\boxed{\omega = \frac{2\pi k}{N}}} = \sum_{n=0}^{N-1} w[n] e^{-j\omega n}$$

$N = 62$

because $w[n]$ is already finite length

$$W(\omega) = \sum_{n=-\infty}^{\infty} w[n] e^{-j\omega n} = \sum_{n=0}^{61} w[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{51} e^{-j\omega n} = \frac{1 - e^{-j\omega 52}}{1 - e^{-j\omega}}$$

$$= e^{-j\omega \frac{51}{2}} \frac{e^{j\omega \frac{52}{2}} - e^{-j\omega \frac{52}{2}}}{e^{j\omega/2} - e^{-j\omega/2}}$$

$$= e^{-j\omega \frac{51}{2}} \frac{\sin(\omega 26)}{\sin(\omega/2)}$$

$$W[k] = W(\omega)\Big|_{\omega = \frac{2\pi k}{62}} = e^{-j \frac{51}{2} \frac{2\pi k}{62}} \frac{\sin\left(\frac{2\pi 26 k}{62}\right)}{\sin\left(\frac{2\pi k}{2 \cdot 62}\right)}$$