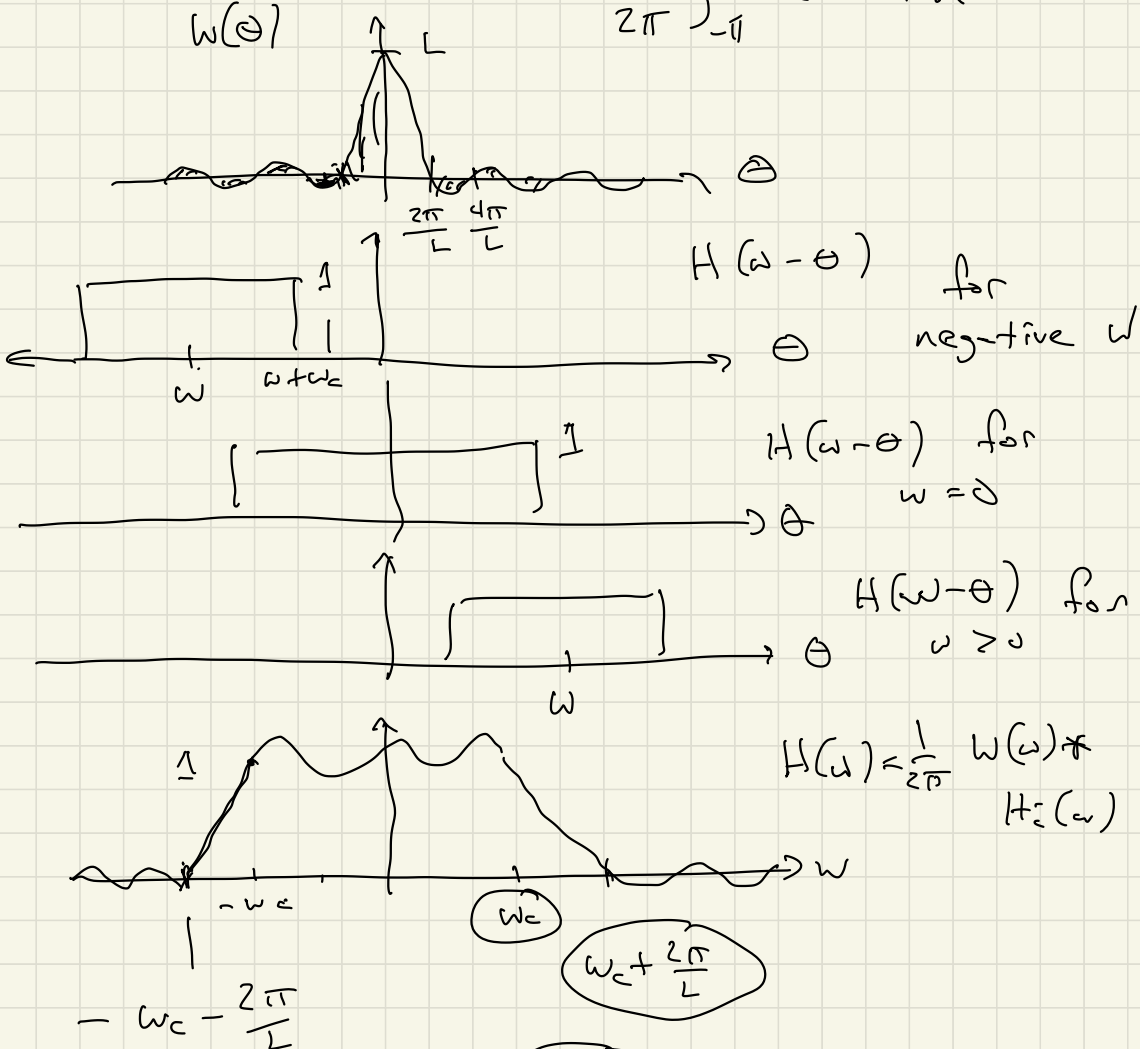
2024 Oct 28

$$H(\omega) = \frac{1}{2\pi} W(\omega) * H_i(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\theta)\, H_i(\omega-\theta)\, d\theta$$

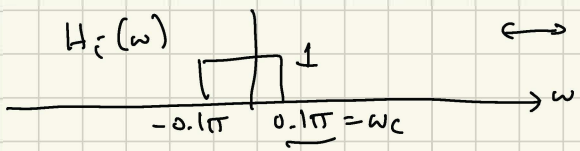
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\omega-\theta)\, H_i(\theta)\, d\theta$$



Transition band: $\frac{2\pi}{L}$

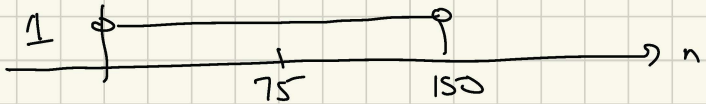
Stopband ripple: related to height of sidelobes

$$h[n] = w[n]\, h_c[n - 75]$$

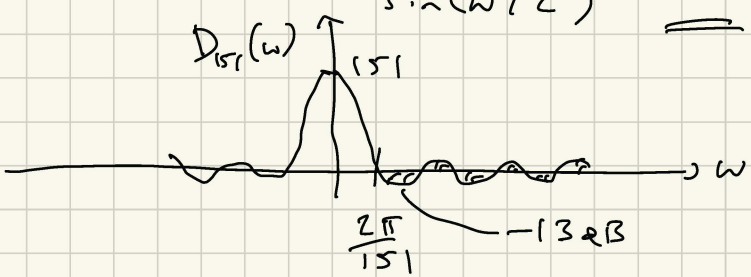


$$\longleftrightarrow \quad h_c[n] = 0.1 \, \text{sinc}(0.1\pi n)$$

$w[n]$ has length 151 samples $= \begin{cases} 1 & 0 \le n \le 150 \\ 0 & \text{else} \end{cases}$



$$W(\omega) = e^{-j\omega 75} \frac{\sin(151\omega/2)}{\sin(\omega/2)} = e^{-j\omega 75} D_{151}(\omega)$$



$$H(\omega) = \frac{1}{2\pi} W(\omega) * H_c(\omega)$$

Areas of sidelobes are $-\frac{4}{3}, \frac{4}{5}, -\frac{4}{7}, \frac{4}{9}, \ldots$

Total area $= -\frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} \cdots$

$$\approx -\frac{20}{15} + \frac{12}{15} = \boxed{-\frac{8}{15} \approx R}$$