

2024 October 21

$$H(\omega) = \begin{cases} 4 & -0.4\pi \leq \omega \leq -0.2\pi \\ 0 & \text{otherwise} \end{cases}$$

$$h[n] = \frac{1}{2\pi} \int_{-\omega}^{\omega} H(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-0.4\pi}^{-0.2\pi} 4 e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \frac{4}{jn} \left[e^{j\omega n} \right]_{-0.4\pi}^{-0.2\pi}$$

$$= \frac{2}{\pi j n} \left[e^{-j0.2\pi n} - e^{-j0.4\pi n} \right]$$

$$= \frac{2}{\pi j n} e^{-j0.3\pi n} \left[e^{j0.1\pi n} - e^{-j0.1\pi n} \right]$$

$$= \frac{2}{\pi j n} e^{-j0.3\pi n} 2j \sin(0.1\pi n)$$

$$= \frac{4}{\pi n} \sin(0.1\pi n) e^{-j0.3\pi n}$$

$$\text{sinc}(0.1\pi n) = \frac{\sin(0.1\pi n)}{0.1\pi n}$$

$$0.1 \text{sinc}(0.1\pi n) = \frac{\sin(0.1\pi n)}{\pi n}$$

$$h[n] = 0.4 \text{sinc}(0.1\pi n) e^{-j0.3\pi n}$$