

2024 Oct 18

IF $x[n]$ SYMMETRIC IN TIME ($x[n] = x[-n]$)

$\Leftrightarrow X_k$ IS REAL

$$X_k = \frac{1}{N_0} \sum_{n=-\left(\frac{N_0-1}{2}\right)}^{\left(\frac{N_0-1}{2}\right)} x[n] e^{-j \frac{2\pi kn}{N_0}}$$

$$= \frac{1}{N_0} \left(x[0] + \sum_{n=1}^{\left(\frac{N_0-1}{2}\right)} x[n] \left(e^{-j \frac{2\pi kn}{N_0}} + e^{j \frac{2\pi kn}{N_0}} \right) \right)$$

↑
Multiplied
by $x[n]$

↑
Multiplied by
 $x[-n]$
 $= x[n]$

$$= \frac{1}{N_0} \left(x[0] + \sum_{n=1}^{\left(\frac{N_0-1}{2}\right)} x[n] 2 \cos \left(\frac{2\pi kn}{N_0} \right) \right)$$

= Real-valued

Example

$$x[n] = \begin{cases} 8 & 0 \leq n \leq 36 \\ 0 & \text{otherwise} \end{cases}$$

$$X(k) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$x[n] = \begin{cases} 8 & 0 \leq n \leq 36 \\ 0 & \text{otherwise} \end{cases}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{36} 8 e^{-j\omega n} = 8 \sum_{n=0}^{36} e^{-j\omega n} = 8 \sum_{n=0}^{37-1} a^n$$

$$= 8 \left(\frac{1-a^{37}}{1-a} \right) = 8 \left(\frac{1-e^{-37j\omega}}{1-e^{-j\omega}} \right)$$

$$= 8 \frac{e^{-j\omega 37/2}}{e^{-j\omega/2}} \left(\frac{e^{j\omega 37/2} - e^{-j\omega 37/2}}{e^{j\omega/2} - e^{-j\omega/2}} \right)$$

$$= 8 e^{-j\omega 36/2} \frac{\sin(\omega 37/2)}{\sin(\omega/2)}$$

$$= 8 e^{-18j\omega} \frac{\sin(\omega 37/2)}{\sin(\omega/2)}$$