

2024 October 14

$$y[n] = \underbrace{18x[n+5]} + \underbrace{18x[n-11]} = \sum_m h[m]x[n-m]$$

$$h[m] = \begin{cases} 18 & m = -5 \\ 18 & m = 11 \\ 0 & \text{else} \end{cases} = \underbrace{18\delta[m+5]} + \underbrace{18\delta[m-11]}$$

$$H(\omega) = \sum_m h[m]e^{-j\omega m} = |H(\omega)|e^{j\angle H(\omega)} = R(\omega)e^{j\phi(\omega)}$$

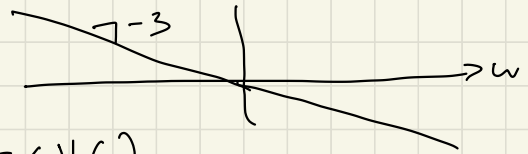
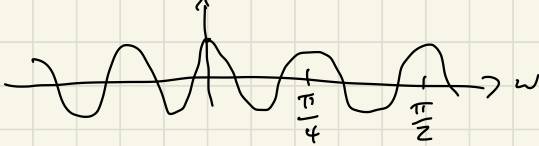
$$= 18e^{j\omega 5} + 18e^{-j\omega 11}$$

$$= 36 \cdot \frac{1}{2} (e^{j\omega 8} + e^{-j\omega 8}) e^{-3j\omega}$$

$$= 36 \cos(8\omega) e^{-3j\omega} = R(\omega)e^{+j\phi(\omega)}$$

$$R(\omega) = 36 \cos(8\omega)$$

$$\phi(\omega) = -3\omega$$



$$R(\omega)e^{j\phi(\omega)} = |H(\omega)|e^{j\angle H(\omega)}$$

$$\angle H(\omega) = \begin{cases} -3\omega & \cos(8\omega) > 0 \\ -3\omega \pm \pi & \cos(8\omega) < 0 \end{cases}$$

$$\angle H(\omega) = \begin{cases} -3\omega \bmod 2\pi & \cos(8\omega) > 0 \\ (-3\omega + \pi) \bmod 2\pi & \cos(8\omega) < 0 \end{cases}$$

$$x \bmod 2\pi \equiv x + k2\pi \quad \text{for } k \text{ s.t. } -\pi \leq x \bmod 2\pi < \pi \quad \boxed{\text{QED}}$$