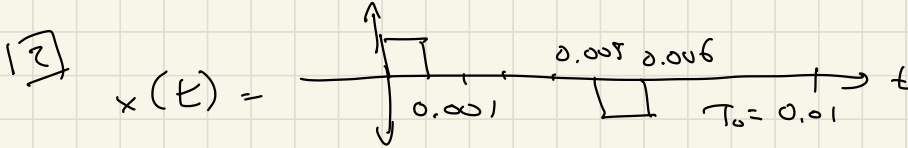


October 4, 2024

Ex-1, 2022

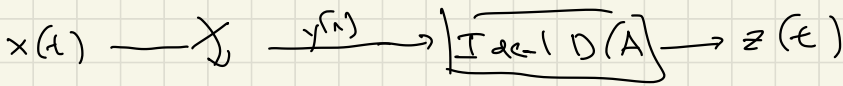
1) $x(t) = -12 \cos(1000\pi t - \frac{\pi}{4}) + 4 \sin(1000\pi t)$
 $= M \cos(1000\pi t + \theta)$



a) find X_k for $t \neq 0$ ($X_0 = 0 !!$)

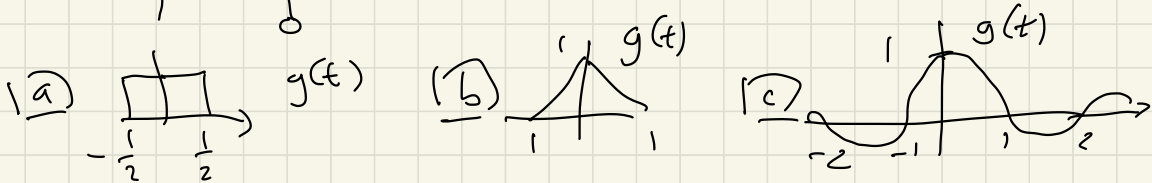
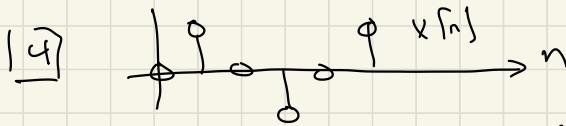
b) $x(t) = \frac{dx}{dt}$ find Y_k

3) $y[n] = x(\frac{n}{F_s})$, $f_s = 10,000$ Hz



a) $x(t) = 3 \cos(2\pi 8000t + \frac{\pi}{4})$

b) $x(t) = 3 \cos(2\pi 12000t + \frac{\pi}{4})$



1) $x(t) = \text{Re}(-12 e^{j1000\pi t} e^{-j\frac{\pi}{4}} + 4 e^{j1000\pi t} e^{-j\frac{\pi}{2}})$
 $= \text{Re}(e^{j1000\pi t} \underbrace{(-12 e^{-j\frac{\pi}{4}} + 4 e^{-j\frac{\pi}{2}})}_z)$

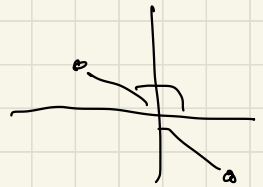
$z = -12 \left(\frac{\sqrt{2}}{2} - j \frac{\sqrt{2}}{2} \right) + 4 \cdot (-j) = -6\sqrt{2} + j(6\sqrt{2} - 4)$

$$z = -6\sqrt{2} + j(6\sqrt{2} - 4)$$

$$|z| = \sqrt{(6\sqrt{2})^2 + (6\sqrt{2} - 4)^2} = M$$

$$\angle z = \theta = \tan^{-1}\left(\frac{6\sqrt{2} - 4}{-6\sqrt{2}}\right) + \pi$$

between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$



2)

1a)

$$X_k = \frac{1}{0.01} \int_0^{0.01} x(t) e^{-j \frac{2\pi k t}{0.01}} dt$$

$$= 100 \left[\int_0^{0.001} e^{-j 200\pi k t} dt - \int_{0.005}^{0.006} e^{-j 200\pi k t} dt \right]$$

$$= \frac{100}{-j 200\pi k} \left[\left[e^{-j 200\pi k t} \right]_0^{0.001} - \left[e^{-j 200\pi k t} \right]_{0.005}^{0.006} \right]$$

$$X_k = \frac{100}{-j 200\pi k} \left[e^{-j 200\pi k \cdot 0.01} - 1 - e^{-j 200\pi k \cdot 0.006} + e^{-j 200\pi k \cdot 0.005} \right]$$

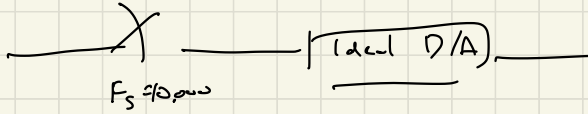
1b) $x(t) = \frac{dy}{dt}$ Find Y_k in terms of X_k

$$X_k = j 2\pi k F_0 Y_k$$

$$\sum X_k e^{j 2\pi k F_0 t} = \frac{d}{dt} \left(\sum Y_k e^{j 2\pi k F_0 t} \right)$$

$$Y_k = \frac{1}{j 2\pi k F_0} X_k$$

$$x(t) = 3 \cos(2\pi 8000t + \frac{\pi}{4})$$



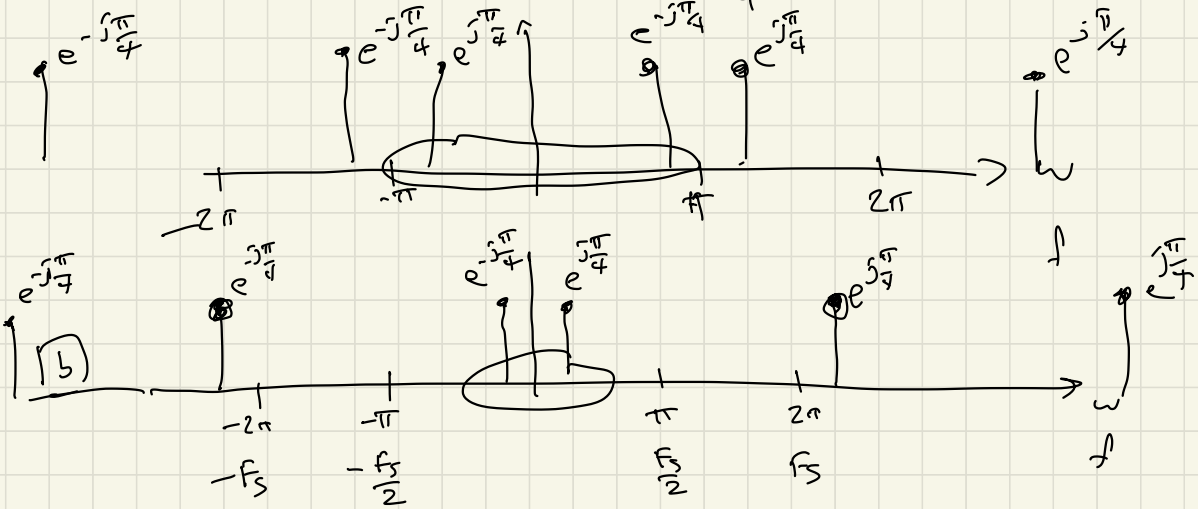
$$\text{a) } x(t) = 3 \cos(2\pi f_a t + \theta_a)$$

$$f_a = nT_s(f, f_s - f, f - f_s, \dots)$$

$$f_a = 10,000 - 8000 = f_s - f$$

$$\theta_a = -\theta = -\frac{\pi}{4}$$

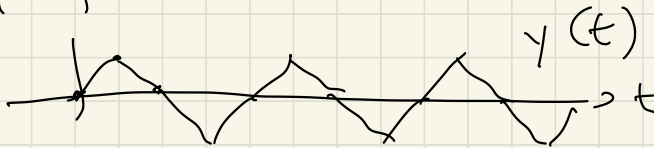
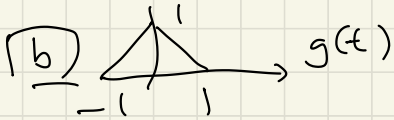
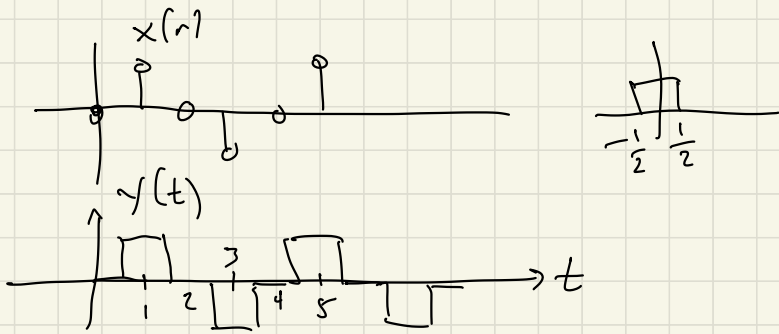
$$z(k) = 3 \cos(2\pi 2000k - \frac{\pi}{4})$$



$$f_a = f - f_s \Rightarrow \theta_a = \theta$$

$$z(t) = 3 \cos(2\pi 2000t + \frac{\pi}{4})$$

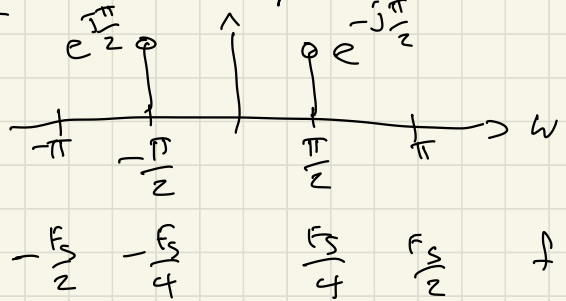
4) a)



c) sinc interpolation: f, z are

just the part of DT spectrum
for $-\pi \leq \omega \leq \pi$

$$x[n] = \sin\left(\frac{\pi n}{2}\right)$$



$$y(t) = \sin\left(2\pi \frac{f_s}{4} t\right)$$

$$f_s = 1 \frac{\text{sample}}{\text{second}}$$

$$y(t) = \sin\left(2\pi \frac{1}{4} t\right)$$

