

2024 November 15

$$y(n) = x[n] + 1.5x[n-1] + x[n-2]$$

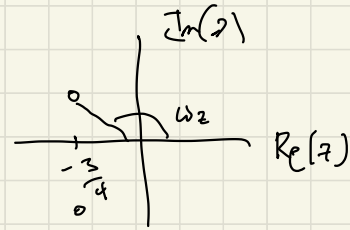
$$H(z) = (1 - e^{j\omega_2} z^{-1})(1 - e^{-j\omega_2} z^{-1}) \Leftarrow$$

$$\rightarrow Y(z) = X(z) + 1.5z^{-1}X(z) + z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 1.5z^{-1} + z^{-2}$$
$$= z^{-2}(z^2 + 1.5z + 1)$$

$$\text{roots}(H(z)) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1.5 \pm \sqrt{(1.5)^2 - 4}}{2}$$

$$4 > (1.5)^2 \Rightarrow \text{complex}$$



$$\text{roots}(H(z)) = -\frac{3}{4} \pm j \frac{1}{2} \sqrt{4 - (1.5)^2} = z_1, z_2$$

$$|z_1| = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{1}{2} \sqrt{4 - \left(\frac{3}{2}\right)^2}\right)^2}$$

$$= \sqrt{\left(\frac{3}{4}\right)^2 + \frac{1}{4} \left(4 - \frac{9}{4}\right)} = 1 \quad \text{by problem statement}$$

$$\omega_2 = \angle z_1 = \tan^{-1} \left( \frac{\frac{1}{2} \sqrt{4 - \left(\frac{3}{2}\right)^2}}{-\frac{3}{4}} \right) \pm \pi$$

$$= \pm \pi + \tan^{-1} \left( \frac{\sqrt{4 - \frac{9}{4}}}{-\frac{3}{2}} \right) = \tan^{-1} \left( \frac{\sqrt{7/4}}{-3/2} \right) \pm \pi$$

$$= \pm \pi + \tan^{-1} \left( \frac{\sqrt{7}/2}{-3/2} \right) = \tan^{-1} \left( \frac{\sqrt{7}}{-3} \right) \pm \pi$$