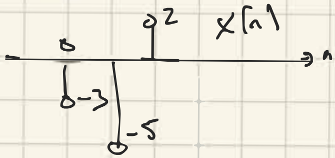
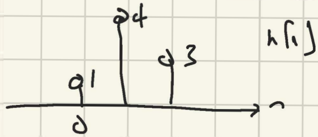
2024 Nov 8

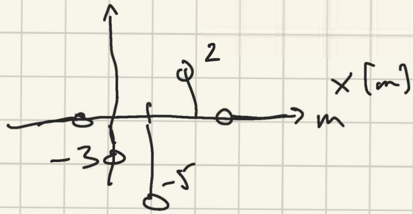
Exam 2, Fall 2023

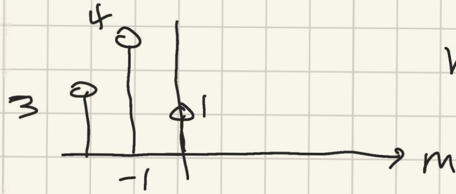
[1]





$y[n] = x[n] * h[n]$





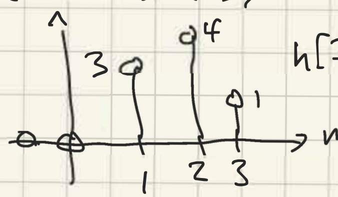
h[n − m] for n = 0

$$y[0] = \sum_m x[m]\,h[0-m]$$
$$= -3$$

$$y[1] = \sum_m x[m]\,h[1-m] = 1\cdot(-5) + 4\cdot(-3) = -17$$

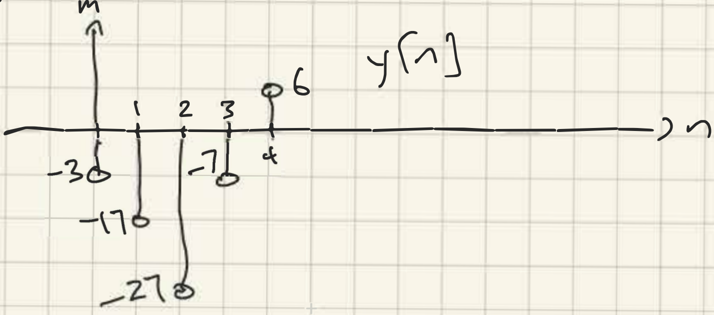
$$y[2] = \sum_m x[m]\,h[2-m] = 1\cdot 2 + 4\cdot(-5) + 3\cdot(-3) = -27$$

$$y[3] = \sum_m x[m]\,h[3-m] = 4\cdot 2 + 3\cdot(-5)$$
$$= -7$$



h[3 − m]

$$y[4] = \sum_m x[m]\,h[4-m] = 3\cdot 2 = 6$$



2

$$y[n] = e^{j(\omega_0 + x[n])n}$$

a) Linear?

$$x_1[n] \longrightarrow y_1[n] = e^{j\omega_0 n} e^{jx_1[n]n}$$

$$x_2[n] \longrightarrow y_2[n] = e^{j\omega_0 n} e^{jx_2[n]n}$$

$$x[n] = x_1[n] + x_2[n] \longrightarrow y[n] = e^{j\omega_0 n} e^{jx[n]n}$$

$$= e^{j\omega_0 n} e^{j(x_1[n] + x_2[n])n}$$

$$= e^{j\omega_0 n} e^{jx_1[n]n} e^{jx_2[n]n}$$

$$= e^{-j\omega_0 n} y_1[n] y_2[n]$$

$\therefore$ $\boxed{\text{NOT LINEAR}}$

b)

$$x[n] = x_1[n-n_0] \longrightarrow y[n] = e^{j\omega_0 n} e^{jx[n]n}$$

$$= e^{j\omega_0 n} e^{jx_1[n-n_0]n} \neq y_1(n-n_0]$$

BUT $$y_1[n-n_0] = e^{j\omega_0(n-n_0)} e^{jx_1[n-n_0]\cdot(n-n_0)}$$

$\therefore$ $\boxed{\text{NOT SHIFT-INVARIANT}}$

c) CAUSAL? Does $y[n]$ depend on any $x[m]$ for $m > n$?

$$y[n] = e^{j\omega_0 n} e^{jx[n]n}$$ depends on $x[m]$ only for $m = n$

$\boxed{\therefore \text{ IT IS CAUSAL}}$

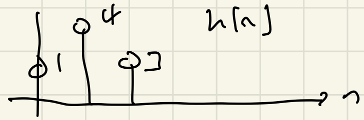
(d) IS IT STABLE?

Def^{n}: $|x[n]| < M_1 \Rightarrow |y[n]| < M_2$

$$y[n] = e^{j\omega_0 n} e^{jx[n]n}$$

$$|y[n]| = |e^{j\omega_0 n} e^{jx[n]n}| = \underline{1}$$

$\boxed{\therefore \text{ IT IS STABLE}}$ (y[n] always bounded)

3) 

$x[n] = e^{j\omega_0 n}$

What is y[n]?

$$e^{j\omega_0 n} \longrightarrow \boxed{\mathcal{H}} \longrightarrow H(\omega_0) e^{j\omega_0 n}$$

$$H(\omega_0) = \sum_n h[n] e^{-j\omega_0 n} = 1 + 4e^{-j\omega_0} + 3e^{-2j\omega_0}$$

$$y[n] = (1 + 4e^{-j\omega_0} + 3e^{-2j\omega_0}) e^{j\omega_0 n}$$

4) (a)

$$F(\omega) = \begin{cases} 1 & |\omega| \leq \frac{\pi}{5} \\ 0 & \text{otherwise} \end{cases}$$

Find f[n].

This is an ideal LPF w/ cutoff $\omega_c = \frac{\pi}{5}$

$$\Rightarrow f[n] = \frac{\omega_c}{\pi} \operatorname{sinc}(\omega_c n) = \frac{1}{5} \operatorname{sinc}\left(\frac{\pi n}{5}\right)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\frac{\pi}{5}}^{\frac{\pi}{5}} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \frac{1}{jn} \left[e^{j\frac{\pi n}{5}} - e^{-j\frac{\pi n}{5}} \right]$$

1b)

$$g[n] = \begin{cases} f[n] & |n| \le 64 \\ 0 & \text{otherwise} \end{cases}$$

$$G(\omega) = \frac{1}{2\pi} F(\omega) * W(\omega) \text{ ; Find } W(\omega).$$

$$g[n] = w[n] f[n] \longleftrightarrow G(\omega) = \frac{1}{2\pi} F(\omega) * W(\omega)$$

$$w[n] = \begin{cases} 1 & |n| \le 64 \\ 0 & \text{otherwise} \end{cases}$$

$$W(\omega) = \sum_{n=-\infty}^{\infty} w[n] e^{-j\omega n} = \sum_{n=-64}^{64} e^{-j\omega n}$$

$$= e^{j\omega 64} \sum_{m=0}^{128} e^{-j\omega m} \qquad m = n + 64$$

$$= e^{j\omega 64} \frac{1 - e^{-j\omega 129}}{1 - e^{-j\omega}} = \cancel{e^{j\omega 64} \frac{e^{-j\omega \frac{129}{2}}}{e^{-j\omega/2}}} \frac{\sin(129\omega/2)}{\sin(\omega/2)}$$

$$= \frac{\sin(\omega N/2)}{\sin(\omega/2)} = \frac{\sin(129\omega/2)}{\sin(\omega/2)}$$

1c) $h(n) = g[n-64] \longleftrightarrow H(\omega) = D(\omega) G(\omega)$

$$\updownarrow$$

$$H(\omega) = e^{-j\omega 64} G(\omega)$$

$$D(\omega) = e^{-j\omega 64}$$