

$$\begin{aligned}
 x(t) &= 0.22 + \\
 &\quad 0.32 \cos(2\pi 421 t + 0.53\pi) \\
 &\quad - 0.22 \sin(2\pi 423 t - 0.92\pi) \\
 &= 0.22 \cos(0) \\
 &+ 0.32 \cos(2\pi 421 t + 0.53\pi) \\
 &- 0.22 \cos(2\pi 423 t - 0.92\pi - \frac{\pi}{2}) \\
 &= \frac{0.22}{\sqrt{2}} e^{j0} \\
 &\quad + \frac{0.32}{\sqrt{2}} e^{j(2\pi 421 t + 0.53\pi)} \\
 &\quad - \frac{0.22}{\sqrt{2}} e^{j(2\pi 423 t - 0.92\pi - \frac{\pi}{2})}
 \end{aligned}$$

$$\begin{aligned}
 &e^{j2\pi f_1 t} + e^{j2\pi f_2 t} + \dots \\
 &= (e^{-j2\pi f_1 t} + e^{j2\pi f_1 t})(e^{-j2\pi f_2 t} + e^{j2\pi f_2 t}) \\
 &= 4 \cos(2\pi f_1 t) \cos(2\pi f_2 t)
 \end{aligned}$$

$$\begin{aligned}
 &+ 0.16 e^{-j2\pi 421 t} \\
 &+ 0.16 e^{j2\pi 423 t} \\
 &- 0.11 e^{-j2\pi 423 t} \\
 &- 0.11 e^{j2\pi 421 t}
 \end{aligned}$$

Beat Tones

$$\begin{aligned}
 &e^{-j2\pi f_1 t} + e^{j2\pi f_1 t} + e^{-j2\pi f_2 t} + e^{j2\pi f_2 t} = \\
 &= 2 \cos(2\pi (f_c + f_b) t) + 2 \cos(2\pi (f_c - f_b) t)
 \end{aligned}$$