Phasors

$$A\cos(2\pi ft+\theta)=\Re\left\{Ae^{j\theta}e^{j2\pi ft}\right\}=\frac{1}{2}e^{-j\theta}e^{-j2\pi ft}+\frac{1}{2}e^{j\theta}e^{j2\pi ft}$$

Spectrum

$$\textbf{Scaling: } y(t)=Gx(t)=\sum_{k=-N}^{N}\left(Ga_k\right)e^{j2\pi f_k t}$$

$$\textbf{Add a Constant: } y(t)=x(t)+C=(a_0+C)+\sum_{k\neq 0}a_k e^{j2\pi f_k t}$$

$$\textbf{Add Signals: } \text{If } f_k=f'_n=f''_m \text{ then } a_k=a'_n+a''_m$$

$$\textbf{Time Shift: } y(t)=x(t-\tau)=\sum_{k=-N}^{N}\left(a_k e^{-j2\pi f_k\tau}\right)e^{j2\pi f_k t}$$

$$\textbf{Frequency Shift: } y(t)=x(t)e^{j2\pi Ft}=\sum_{k=-N}^{N}a_k e^{j2\pi(f_k+F)t}$$

$$\textbf{Differentiation: } y(t)=\frac{dx}{dt}=\sum_{k=-N}^{N}\left(j2\pi f_k a_k\right)e^{j2\pi f_k t}$$

Fourier Series

$$\textbf{Analysis: } X_k=\frac{1}{T_0}\int_0^{T_0}x(t)e^{-j2\pi kt/T_0}dt$$

$$\textbf{Synthesis: } x(t)=\sum_{k=-\infty}^{\infty}X_k e^{j2\pi kt/T_0}$$

Sampling and Interpolation:

$$x[n]=x\left(t=\frac{n}{F_s}\right)$$

$$f_a=\min\left(f \bmod F_s, -f \bmod F_s\right)$$

$$z_a=\begin{cases}z & f \bmod F_s < -f \bmod F_s\\ z^* & f \bmod F_s > -f \bmod F_s\end{cases}$$

$$y(t)=\sum_{n=-\infty}^{\infty}y[n]p(t-nT_s)$$