UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 401 Signal and Image Analysis
Spring 2017

EXAM 2

Tuesday, March 28, 2017

- This is a CLOSED BOOK exam. You may use one sheet (front and back) of hand-written notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- There are a total of 40 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
Total	

Name: _____

Problem 1 (10 points)

A particular system adds together a signal, 90% of its own echo 3 samples later, and 80% of its own echo 10 samples later:

$$y[n] = x[n] + 0.9x[n-3] + 0.8x[n-10]$$

What is the impulse response, $h[n]$, of this system?

Problem 2 (10 points)

Short-time energy, $y[n]$, is computed by averaging N consecutive squared samples of a signal $x[n]$, where N is some arbitrary constant:

$$y[n] = \frac{1}{N} \sum_{m=0}^{N-1} x^2[n-m]$$

(a) Is this system linear? Prove your answer, by determining whether or not $x_3[n] = ax_1[n] + bx_2[n]$ produces the output $y_3[n] = ay_1[n] + by_2[n]$.

(b) Is this system time-invariant? Prove your answer, by determining whether or not $x_2[n] = x_1[n - n_0]$ produces the output $y_2[n] = y_1[n - n_0]$.

Problem 3 (10 points)

A periodic continuous-time signal has the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0}$$

Suppose that $T_0 = 0.001$s. Suppose that $x(t)$ is lowpass filtered by an ideal anti-aliasing filter with a cutoff of 3kHz, then sampled at $F_s = 6$kHz to create $x[n]$. $x[n]$ is then passed through a 3-sample averager to create $y[n]$:

$$y[n] = \frac{1}{3} \sum_{m=0}^{2} x[n-m]$$

The signal $y[n]$ is sent through an ideal D/A with the same sampling frequency, $F_s = 10$kHz, to create the signal $y(t)$, which can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j2\pi kt/T_0}$$

(a) For which values of k does $Y_k = 0$, either because of the anti-aliasing filter or because of the digital filter?

(b) Find the amplitudes of Y_k in terms of X_k for all k, including $k = 0$. You don't need to worry about phase.

Problem 4 (10 points)

Consider the following ideal bandpass filter:

$$H(\omega) = \begin{cases} 1 & \frac{\pi}{6} < |\omega| < \frac{\pi}{3} \\ 0 & \text{otherwise} \end{cases}$$

Find the impulse response, $h[n]$.