

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 401 SIGNAL AND IMAGE ANALYSIS
Spring 2024

EXAM 2

Monday, November 11, 2024

- This is a **CLOSED BOOK** exam.
- You are permitted one sheet of handwritten notes, 8.5x11.
- Calculators and computers are not permitted.
- Do not leave any unresolved integrals or summations in your answer. Beyond that point, you do not need to simplify your answer unless simplification is explicitly requested.
- If you're taking the exam online, you will need to have your webcam turned on. Your exam will appear on Gradescope at exactly 1:00pm; you will need to photograph and upload your answers by exactly 2:00pm.
- There will be a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must **SHOW YOUR WORK** to get full credit.

Name: _____

Convolution

$$h[n] * x[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

Linearity, Shift-Invariance, Causality, Stability

- a) **Linearity:** $x_1[n] + x_2[n] \rightarrow y_1[n] + y_2[n]$
- b) **Shift-Invariance:** $x_1[n - n_0] \rightarrow y_1[n - n_0]$
- c) **Causality:** $h[n] = 0$ for $n < 0$
- d) **Stability:** $|x[n]| < M_1 \Rightarrow |y[n]| < M_2$

Frequency Response

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$
$$h[n] * \cos(\omega n) = |H(\omega)| \cos(\omega n + \angle H(\omega))$$

DTFT Properties

- a) Periodicity: $X(\omega + 2\pi) = X(\omega)$
- b) Linearity: $z[n] = ax[n] + by[n] \leftrightarrow Z(\omega) = aX(\omega) + bY(\omega)$
- c) Time Shift: $x[n - n_0] \leftrightarrow e^{-j\omega n_0} X(\omega)$
- d) Frequency Shift: $e^{j\omega_0 n} x[n] \leftrightarrow X(\omega - \omega_0)$
- e) Convolution in Time: $y[n] = h[n] * x[n] \leftrightarrow Y(\omega) = H(\omega)X(\omega)$
- f) Convolution in Frequency: $y[n] = w[n]x[n] \leftrightarrow Y(\omega) = \frac{1}{2\pi} W(\omega) * X(\omega)$

Rectangular Window and Ideal LPF

$$\sum_{n=0}^{L-1} a^n = \frac{1 - a^L}{1 - a}$$
$$w_R[n] = \begin{cases} 1 & 0 \leq n \leq N - 1 \\ 0 & \text{otherwise} \end{cases} \leftrightarrow W_R(\omega) = e^{-\frac{j\omega(N-1)}{2}} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$
$$h_{\text{ideal}}[n] = \frac{\omega_c}{\pi} \text{sinc}(\omega_c n) \leftrightarrow H_{\text{ideal}}(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Hamming Window

$$w_H[n] = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N - 1 \\ 0 & \text{otherwise} \end{cases}$$
$$W_H(\omega) = 0.54W_R(\omega) + 0.23W_R\left(\omega - \frac{2\pi}{N-1}\right) + 0.23W_R\left(\omega + \frac{2\pi}{N-1}\right)$$

1. (28 points) Consider a system whose output, $y[n]$, is an average of the samples of its input:

$$y[n] = \sum_{m=m_1}^{m_2} x[n-m]$$

The behavior of the system depends on the values of m_1 and m_2 . You may assume that $m_1 < m_2$, but you should consider all possibilities in the range $-\infty \leq m_1 < m_2 \leq \infty$.

- a) Are there any values of m_1 or m_2 for which this system is nonlinear? If so, list all values of m_1 and m_2 for which the system is nonlinear. If not, prove that the system is linear.

- b) Are there any values of m_1 or m_2 for which this system is shift-varying? If so, list all values of m_1 and m_2 for which the system is shift-varying. If not, prove that the system is shift-invariant.

c) Are there any values of m_1 or m_2 for which this system is non-causal? If so, list all values of m_1 and m_2 for which the system is non-causal. If not, prove that the system is causal.

d) Are there any values of m_1 or m_2 for which this system is unstable? If so, list all values of m_1 and m_2 for which the system is unstable. If not, prove that the system is stable.

2. (24 points) Suppose the input to a system is $x[n] = u[n] - u[n - L]$, and the impulse response is $h[n] = a^n u[n]$, where $u[n]$ is the unit step function ($u[n] = 1$ for $n \geq 0$, $u[n] = 0$ for $n < 0$). Find $x[n] * h[n]$ for all n as a function of a and L .

3. (24 points) Suppose that the frequency response of a system is:

$$H(\omega) = \begin{cases} 67 & -\frac{\pi}{5} \leq \omega \leq \frac{2\pi}{5} \\ 0 & \text{otherwise} \end{cases}$$

Find the value of $h[n]$, the impulse response of this system, for all n .

4. (24 points) Consider the following signal:

$$w[n] = \begin{cases} 0.1 & -27 \leq n < -12 \\ 0 & \text{otherwise} \end{cases}$$

Note that the DTFT of this signal can be expressed as $W(\omega) = e^{j\omega b}R(\omega)$ for some real-valued scalar b and real-valued function $R(\omega)$. Find b and $R(\omega)$.

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