UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 401 Signal Processing
EXAM 1

October 11, 2024

- This is a CLOSED BOOK exam.
- You are permitted one sheet of handwritten notes, 8.5x11.
- Calculators and computers are not permitted.
- Do not leave any unresolved integrals or summations in your answer. Other than that, however, you should not simplify explicit numerical expressions.
- If you're taking the exam online, you will need to have your webcam turned on. Your exam will appear on Gradescope at exactly 1:00pm; you will need to photograph and upload your answers by exactly 2:00pm.
- There will be a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Name: ______________________________

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Phasors

$$A\cos(2\pi ft+\theta)=\Re\left\{Ae^{j\theta}e^{j2\pi ft}\right\}=\frac{1}{2}e^{-j\theta}e^{-j2\pi ft}+\frac{1}{2}e^{j\theta}e^{j2\pi ft}$$

Spectrum

$$\textbf{Scaling:}\;\; y(t)=Gx(t)=\sum_{k=-N}^{N}(Ga_k)\,e^{j2\pi f_k t}$$

$$\textbf{Add a Constant:}\;\; y(t)=x(t)+C=(a_0+C)+\sum_{k\neq 0}a_k e^{j2\pi f_k t}$$

$$\textbf{Add Signals:}\;\; \text{If } f_k=f'_n=f''_m \;\text{ then }\; a_k=a'_n+a''_m$$

$$\textbf{Time Shift:}\;\; y(t)=x(t-\tau)=\sum_{k=-N}^{N}\left(a_k e^{-j2\pi f_k\tau}\right)e^{j2\pi f_k t}$$

$$\textbf{Frequency Shift:}\;\; y(t)=x(t)e^{j2\pi Ft}=\sum_{k=-N}^{N}a_k e^{j2\pi(f_k+F)t}$$

$$\textbf{Differentiation:}\;\; y(t)=\frac{dx}{dt}=\sum_{k=-N}^{N}(j2\pi f_k a_k)\,e^{j2\pi f_k t}$$

Fourier Series

$$\textbf{Analysis:}\;\; X_k=\frac{1}{T_0}\int_0^{T_0}x(t)e^{-j2\pi kt/T_0}dt$$

$$\textbf{Synthesis:}\;\; x(t)=\sum_{k=-\infty}^{\infty}X_k e^{j2\pi kt/T_0}$$

Sampling and Interpolation:

$$x[n]=x\left(t=\frac{n}{F_s}\right)$$

$$f_a=\min\left(f \bmod F_s,-f \bmod F_s\right)$$

$$z_a=\begin{cases} z & f \bmod F_s< -f \bmod F_s\\ z^* & f \bmod F_s> -f \bmod F_s\end{cases}$$

$$y(t)=\sum_{n=-\infty}^{\infty}y[n]p(t-nT_s)$$

1. (25 points) Suppose that

$$\begin{aligned} x(t) &= 2\cos(2\pi 400t) + 5\sin(2\pi 1000t) \\ y(t) &= 5\cos(2\pi 1000t) + 2\sin(2\pi 2000t) \\ z(t) &= x(t) + y(t) \end{aligned}$$

What is the spectrum of $z(t)$? Recall that a spectrum is defined to be a list of frequencies with their corresponding phasors, including both positive and negative frequencies.

2. (25 points) Consider the following periodic signal:

$$x(t) = \begin{cases} 5e^{-400t} & 0 \leq t < 0.01 \\ x(t - 0.01) & \text{otherwise} \end{cases}$$

Find the Fourier series coefficients of this signal, X_k, including X_0. The only variable appearing in your answer should be k, but your answer may also include numerical constants such as e, π, and j.

3. (25 points) Consider the signal

$$x_a(t) = \cos\left(2\pi 3300t + \frac{\pi}{6}\right) + \cos\left(2\pi 7300t + \frac{\pi}{4}\right) + \cos\left(2\pi 8300t + \frac{\pi}{3}\right)$$

Suppose that this signal is sampled at 8000 samples/second, to create the signal $x[n]$:

$$x[n] = \cos(\omega_1 n + \theta_1) + \cos(\omega_2 n + \theta_2) + \cos(\omega_3 n + \theta_3)$$

Find values of ω_k and θ_k, $k \in \{1, 2, 3\}$, such that $0 \leq \omega_k \leq \pi$ and $x[n] = x_a(n/8000)$.

4. (25 points) Consider the following discrete-time signal, $x[n]$:

$$x[n] = \sin\left(\frac{2\pi n}{8}\right),$$

thus $x[n]$ is periodic with a period of 8, and its first 8 samples are $\left\{0, \frac{\sqrt{2}}{2}, 1, \frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}, -1, -\frac{\sqrt{2}}{2}\right\}$. Suppose that $x_a(t) = \sum_{n=-\infty}^{\infty} x[n]p(t-nT)$, where $T = 0.001$ seconds, and $p(t)$ is a triangular interpolation kernel:

$$p(t) = \max\left(1 - 1000|t|, 0\right)$$

(a) (15 points) Sketch the period of $x_a(t)$ for $0 \leq t \leq 0.008$, or else specify its values using an equation.

(b) (10 points) What frequencies are present in the spectrum of $x_a(t)$?

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