UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 401 Signal and Image Analyais
Spring 2023

EXAM 2

Monday, October 30, 2023

- This is a CLOSED BOOK exam.

- You are permitted one sheet of handwritten notes, 8.5x11.

- Calculators and computers are not permitted.

- Do not simplify explicit numerical expressions. The expression "$e^{-5}\cos(3)$" is a MUCH better answer than "-0.00667".

- If you're taking the exam online, you will need to have your webcam turned on. Your exam will appear on Gradescope at exactly 1:00pm; you will need to photograph and upload your answers by exactly 2:00pm.

- There will be a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.

- You must SHOW YOUR WORK to get full credit.

Name: _____

Convolution

$$h[n] * x[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

Frequency Response

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

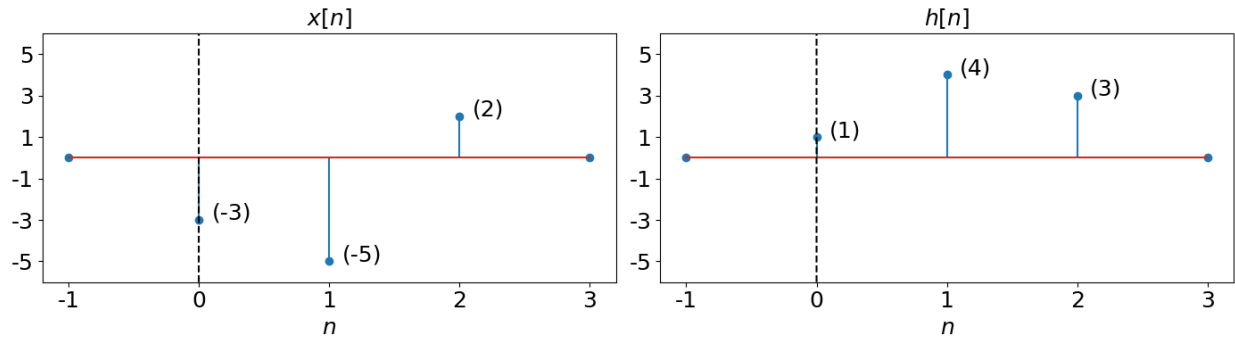$$h[n] * \cos(\omega n) = |H(\omega)|\cos\left(\omega n + \angle H(\omega)\right)$$

Rectangular Window and Ideal LPF

$$w_R[n] = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \leftrightarrow W_R(\omega) = e^{-\frac{j\omega(N-1)}{2}} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

$$h_{\text{ideal}}[n] = \frac{\omega_c}{\pi}\text{sinc}(\omega_c n) \leftrightarrow H_{\text{ideal}}(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Hamming Window

$$w_H[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

$$W_H(\omega) = 0.54 W_R(\omega) + 0.23 W_R\left(\omega - \frac{2\pi}{N-1}\right) + 0.23 W_R\left(\omega + \frac{2\pi}{N-1}\right)$$

1. (19 points) The following figure shows two signals, $x[n]$ and $h[n]$. Each signal has three nonzero samples, with values as shown in the figure.

Find $y[n] = h[n] * x[n]$. Plot your answer using a stem plot similar to the plot shown above, and label the value of each nonzero sample in parentheses next to the sample, as shown above.

2. (32 points) Frequency modulation (FM) is a system that accepts an input signal $x[n]$, and computes an output signal $y[n]$ according to

$$y[n] = e^{j(\omega_0 + x[n])n},$$

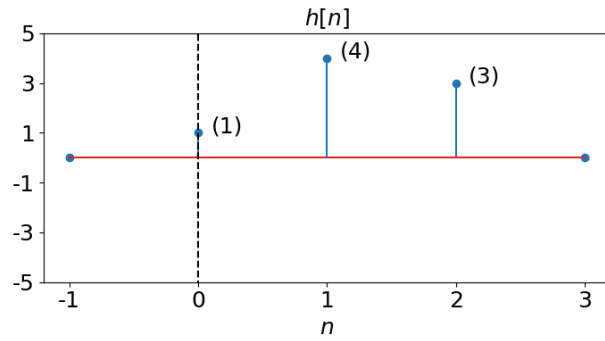where ω_0 is a real scalar constant called the carrier frequency.

(a) Is FM linear? Prove your answer.

(b) Is FM shift-invariant? Prove your answer.

(c) Is FM causal? Prove your answer.

(d) Is FM stable? Prove your answer.

3. (19 points) Suppose the signal $x[n] = e^{j\omega_0 n}$ is passed through a linear shift-invariant system with the following impulse response:

$h[n]$

Find the system output. Your answer should be a function of only n and ω_0.

4. (30 points) This problem is about the design of a lowpass filter.

(a) The desired frequency response is

$$F(\omega) = \begin{cases} 1 & |\omega| \leq \frac{\pi}{5} \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

What is $f[n]$?

(b) As a first step toward implementation, the filter is truncated, thus

$$g[n] = \begin{cases} f[n] & |n| \leq 64 \\ 0 & \text{otherwise} \end{cases}$$

The resulting frequency response can be writtten as

$$G(\omega) = \frac{1}{2\pi} F(\omega) * W(\omega), \qquad (2)$$

where $F(\omega)$ is the function shown in Eq. (1), and $W(\omega)$ is some function of ω. Find $W(\omega)$ as a function of ω.

(c) As a second step toward implementation, the filter is delayed by 64 samples, thus

$$h[n] = g[n - 64]$$

The resulting frequency response can be written as $H(\omega) = D(\omega)G(\omega)$, where $G(\omega)$ is the function shown in Eq. (2), and $D(\omega)$ is some function of ω. Find $D(\omega)$ as a function of ω.

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