UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498DLJ Principles of Signal Analysis
Fall 2011

MIDTERM EXAM SOLUTIONS

Friday, October 21, 2011

- This is a CLOSED BOOK exam.
- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: _____

Problem 1 (20 points)

Calculate the Fourier series coefficients a_0, a_k, b_k, $k = 1, 2, \ldots$, for the periodic signal $x(t) = x(t+8)$:

$$x(t) = \begin{cases} 1, & 0 \le t < 1 \\ -1, & 1 \le t \le 3 \\ 0, & 3 < t < 8 \end{cases}$$

Solutions:

$$X_0 = a_0 = -\frac{1}{8}$$

$$X_k = a_k - jb_k = \frac{1}{-j2\pi k}\left(2e^{-j\frac{\pi k}{4}} - 1 - e^{-j\frac{3\pi k}{4}}\right)$$

Problem 2 (20 points)

Compute the Fourier transform of $y(t)$. Write your answer in the form $Y(\Omega) = jA(\Omega)$, for some real-valued function $A(\Omega)$. The function $A(\Omega)$ should have no complex exponentials in it, and no imaginary parts.

$$y(t) = \begin{cases} 0.25, & -1 \leq t < 0 \\ -0.25, & 0 \leq t < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Problem 3 (20 points)

Start with the signal $x[n] = \delta[n] + \delta[n-1]$. Find the 4-point DFT, $X[k]$, for $0 \leq k \leq 3$. Simplify, so that there are no complex exponentials left in your answer.

Problem 4 (20 points)

Suppose that we have a signal bandlimited to 5kHz. We want to digitally bandpass filter it to pass all signal components in the range $1000 \leq f \leq 2000$Hz, where $\Omega = 2\pi f$, and to eliminate all other frequencies.

(a) What is the minimum F_s necessary to avoid aliasing?

Solution:

$$F_s = 10\text{kHz}$$

(b) For the sampling rate F_s that you chose in part (a), what are the corresponding bandpass edges, ω_l and ω_u, of the discrete-time filter $H_d(\omega)$?

(c) Sketch the frequency response $H_d(\omega)$ of the desired filter, for $0 \leq \omega \leq 2\pi$ (note the non-standard frequency range over whch I have asked you to sketch the frequency response!!)

Problem 5 (20 points)

Assume that $x[n] = x_c(nT)$, where $1/T = 10,000$ samples/second. For each of the following signals, find $x[n]$ and $X_d(\omega)$.

(a) $x_c(t) = 0.2\text{sinc}(2000\pi t)$ (sinc, not sin!)

(b) $y_c(t) = \cos(7000\pi t)$.

Solution:

$X_d(\omega)$ is a spectrum having the (frequency,phasor) pairs $\{(-0.6\pi, \frac{1}{2}), (0.6\pi, \frac{1}{2})\}$.