1 $x[n] = \delta[n-15] + \delta[n-30]$

a $X[k]$ HAS $N = 32$

$$X[k] = \sum_{n=0}^{31} x[n] e^{-j\frac{2\pi k n}{32}}$$

$$= e^{-j\frac{2\pi k 15}{32}} + e^{-j\frac{2\pi k 30}{32}}$$

b $$h[n] = \begin{cases} e^{-n/14} & 0 \le n \le 14 \\ 0 & \text{otherwise} \end{cases}$$

$h[n] \longleftrightarrow H[k]$

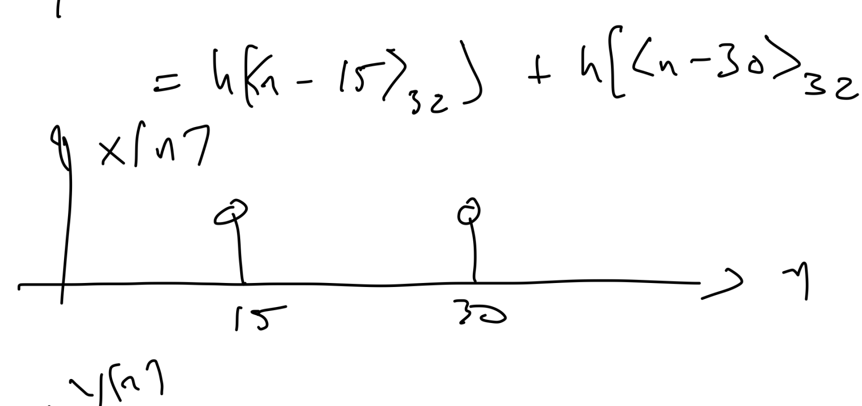
$Y[k] = H[k] X[k]$

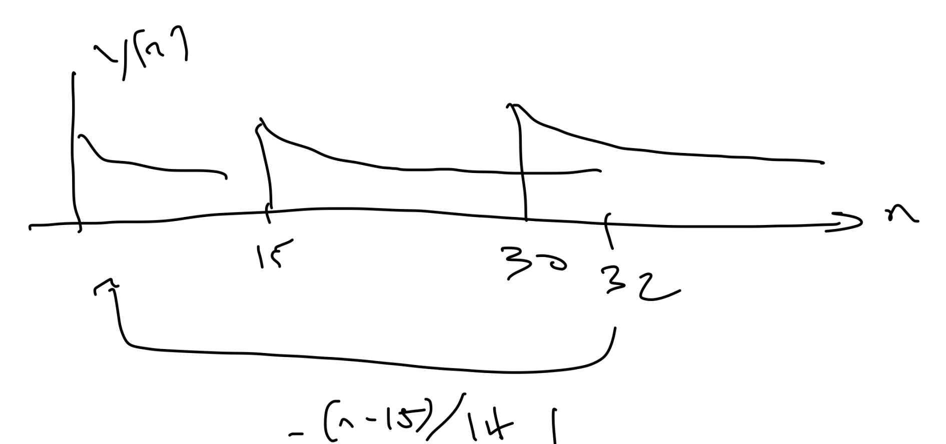
$y[n] \longleftrightarrow Y[k]$

FIND $y[n]$

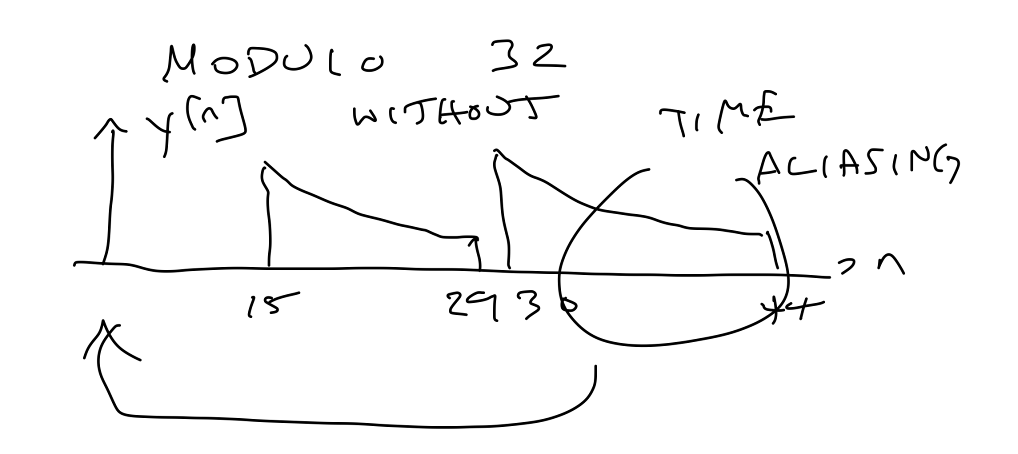
$y[n] = x[n] \circledast h[n]$

$= h[\langle n-15 \rangle_{32}] + h[\langle n-30 \rangle_{32}]$

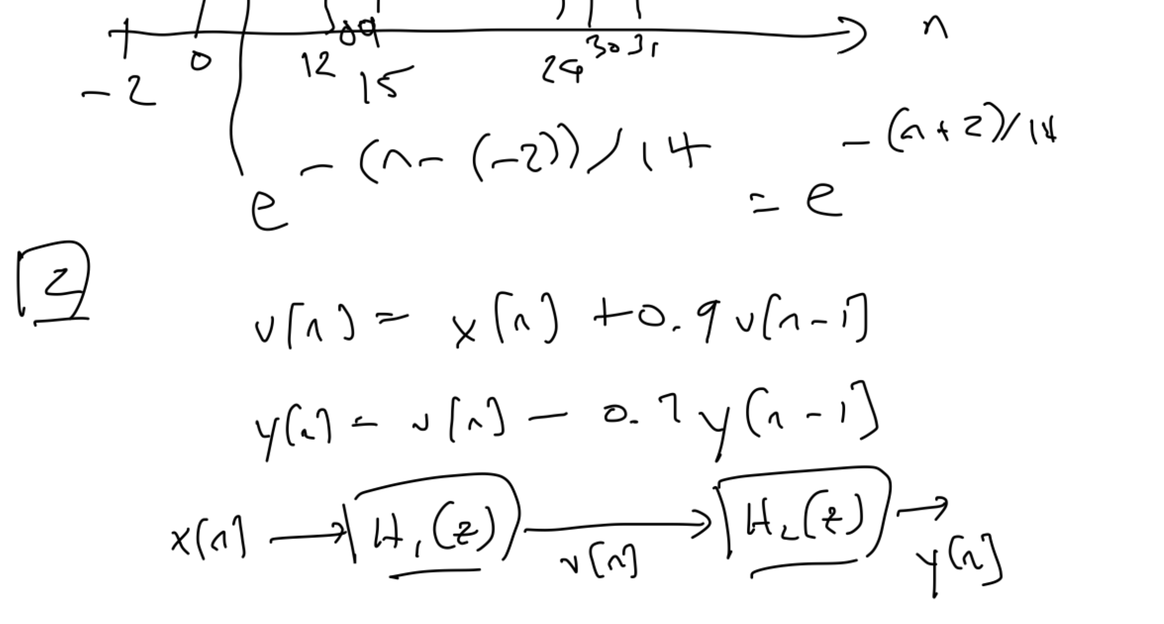




$$y[n] = e^{-(n-15)/14} \Big|_{15 \le n \le 29} + e^{-(n-30)/14} \Big|_{30 \le n \le 44}$$



$$y[n] = \begin{cases} e^{-(n-15)/14} & 15 \le n \le 29 \\ e^{-(n-30)/14} & 30 \le n \le 31 \\ e^{-(n+2)/14} & 0 \le n \le 12 \\ 0 & 13 \le n \le 14 \end{cases}$$



2

$v[n] = x[n] + 0.9 v[n-1]$

$y[n] = v[n] - 0.7 y[n-1]$

a $H(z) = H_1(z) H_2(z)$

$V(z) = X(z) + 0.9 z^{-1} V(z)$

$V(z)(1 - 0.9z^{-1}) = X(z)$

$$\frac{V(z)}{X(z)} = \frac{1}{1 - 0.9 z^{-1}}$$

$Y(z) = V(z) - 0.7 z^{-1} Y(z)$

$Y(z)(1 + 0.7z^{-1}) = V(z)$

$$\frac{Y(z)}{V(z)} = \frac{1}{1 + 0.7 z^{-1}}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{(1 - 0.9z^{-1})(1 + 0.7z^{-1})}$$

b

$$H(z) = \frac{C_1}{1 - 0.9z^{-1}} + \frac{C_2}{1 + 0.7z^{-1}}$$

$$= \frac{1}{(1 - 0.9z^{-1})(1 + 0.7z^{-1})}$$

$C_1(1 + 0.7z^{-1}) + C_2(1 - 0.9z^{-1}) = 1$

EVALUATE AT: $z = -0.7$

$$\Rightarrow C_2\left(1 - \frac{0.9}{(-0.7)}\right) = 1$$

$$C_2\left(1 + \frac{9}{7}\right) = 1$$

$$C_2 = \frac{1}{1 + \frac{9}{7}} = \frac{7}{16}$$

EVALUATE AT: $z = 0.9$

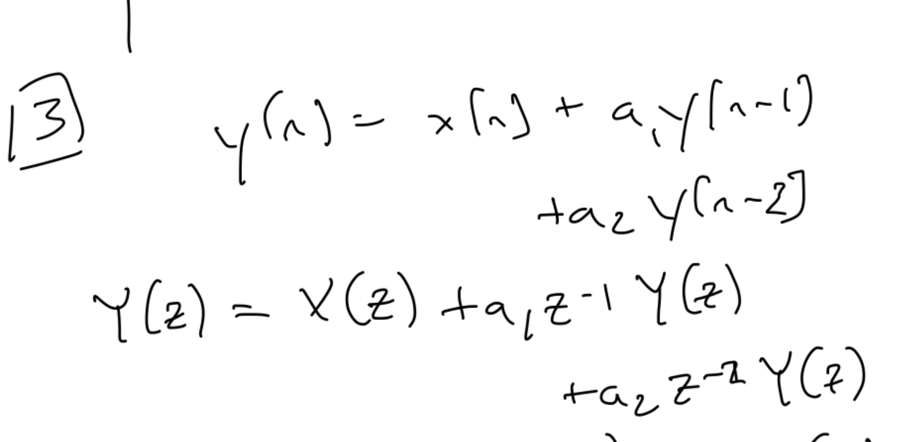
$$C_1\left(1 + \frac{0.7}{0.9}\right) = 1$$

$$C_1\left(1 + \frac{7}{9}\right) = 1$$

$$C_1 = \frac{1}{1 + \frac{7}{9}} = \frac{9}{16}$$

$$H(z) = \frac{9/16}{1 - 0.9z^{-1}} + \frac{7/16}{1 + 0.7z^{-1}}$$

$$h[n] = \frac{9}{16}(0.9)^n u[n] + \frac{7}{16}(-0.7)^n u[n]$$



3 $y[n] = x[n] + a_1 y[n-1] + a_2 y[n-2]$

$Y(z) = X(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z)$

$Y(z)(1 - a_1 z^{-1} - a_2 z^{-2}) = X(z)$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

$$\omega_1 \left[\frac{\text{RADIANS}}{\text{SAMPLE}}\right] = 2\pi \left[\frac{\text{RADIANS}}{\text{CYCLE}}\right] \frac{440 \left[\frac{\text{CYCLES}}{\text{SECOND}}\right]}{10{,}000 \left[\frac{\text{SAMPLES}}{\text{SECOND}}\right]}$$

$$\omega_1 = \frac{2\pi 440}{10{,}000}$$

$$\sigma_1 = \frac{2\pi 20}{10{,}000}$$

$p_1 = e^{-\sigma_1 + j\omega_1}$

$p_1^* = e^{-\sigma_1 - j\omega_1}$

$(1 - p_1 z^{-1})(1 - p_1^* z^{-1})$

$= 1 - (p_1 + p_1^*) z^{-1} + p_1 p^* z^{-2}$

$= 1 - 2e^{-\sigma_1} \cos(\omega_1) z^{-1} + e^{-2\sigma_1} z^{-2}$

$$a_1 = 2e^{-\frac{2\pi 20}{10{,}000}} \cos\left(\frac{2\pi 440}{10{,}000}\right)$$

$$a_2 = -e^{-\frac{4\pi 20}{10{,}000}}$$

b $x[n] = \delta[n] \Longrightarrow y[n] = h[n]$

$$H(z) = \frac{1}{(1 - p_1 z^{-1})(1 - p_1^* z^{-1})}$$

$$= \frac{C_1}{1 - p_1 z^{-1}} + \frac{C_2}{1 - p_1^* z^{-1}}$$

$1 = C_1(1 - p_1^* z^{-1}) + C_2(1 - p_1 z^{-1})$

EVAL @ $z = p_1$

$$1 = C_1\left(1 - \frac{p_1^*}{p_1}\right)$$

$$\frac{1}{1 - \frac{p_1^*}{p_1}} = C_1 = \frac{p_1}{p_1 - p_1^*}$$

$$= \frac{e^{-\sigma_1 + j\omega_1}}{e^{-\sigma_1 + j\omega_1} - e^{-\sigma_1 - j\omega_1}}$$

$$= \frac{e^{+j\omega_1}}{e^{j\omega_1} - e^{-j\omega_1}} = \frac{e^{j\omega_1}}{2j\sin(\omega_1)}$$

$$h[n] = C_1 p_1^n u[n] + C_1^* (p_1^*)^n u[n]$$

$$= \frac{e^{j\left(\frac{2\pi 440}{10{,}000}\right)}}{2j\sin\left(\frac{2\pi 440}{10{,}000}\right)} \left(e^{-\sigma_1 + j\omega_1}\right)^n u[n]$$

$$+ \frac{e^{-j\left(\frac{2\pi 440}{1000}\right)}}{-2j\sin\left(\frac{2\pi 440}{10{,}000}\right)} \left(e^{-\sigma_1 - j\omega_1}\right)^n u[n]$$

WHERE $\sigma_1 = \frac{2\pi 20}{10{,}000}$, $\omega_1 = \frac{2\pi 440}{10{,}000}$

$$h[n] = \frac{1}{\sin(\omega_1)} e^{-\sigma_1 n} \sin(\omega_1 (n+1)) u[n]$$