Fall 2022 Exam 2

$\boxed{1}$

$$y[n] = \begin{cases} x[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$\boxed{a}$ Linear?

$$x_1[n] \longrightarrow y_1[n] = \begin{cases} x_1[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$x_2[n] \longrightarrow y_2[n] = \begin{cases} x_2[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$y_1[n] + y_2[n] = \begin{cases} x_1[n] + x_2[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$x_3[n] = x_1[n] + x_2[n] \longrightarrow y_3[n] = \begin{cases} x_3[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$= \begin{cases} x_1[n] + x_2[n] & \text{even} \\ 0 & \text{odd} \end{cases}$$

$$= y_1[n] + y_2[n]$$

∴ LINEAR

$\boxed{b}$ SHIFT-INVARIANT?

$$x_1[n] \longrightarrow y_1[n] = \begin{cases} x_1[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$y_1[n-1] = \begin{cases} x_1[n-1] & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

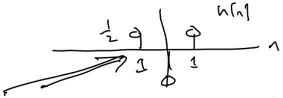
$$x_2[n] = x_1[n-1] \longrightarrow y_2[n] = \begin{cases} x_2[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$y_2[n] = \begin{cases} x_1[n-1] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$\neq y_1[n-1]$$

∴ NOT SHIFT-INVARIANT

$\boxed{2}$



$$h[n] = -\delta[n] + \tfrac{1}{2}\left(\delta[n-1] + \delta[n+1]\right)$$

$\boxed{a}$ CAUSAL? No Because h[n] not right-sided ($h[-1] \neq 0$)

$\boxed{b}$ STABLE?

$$\left(\begin{array}{l} \sum_n |h[n]| \quad \text{FINITE ?} \\ \sum_n |h[n]| = \tfrac{1}{2} + 1 + \tfrac{1}{2} \quad \text{IS FINITE} \end{array} \right)$$

$\Rightarrow$ STABLE

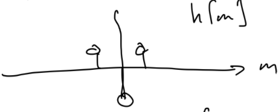
PUT IN ANY $x[n]$ SUCH THAT $|x[n]| \leq M$, M FINITE

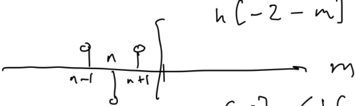
$$\Rightarrow \left(\begin{array}{l} y[n] = +\tfrac{1}{2} x[n-1] + \tfrac{1}{2} x[n+1] - x[n] \\ |y[n]| \leq \tfrac{1}{2} M + \tfrac{1}{2} M + M \\ \quad = 2M \quad \text{FINITE} \end{array} \right.$$

$\Rightarrow$ STABLE

$\boxed{c}$ $h[n] * h[n] = g[n]$

$g[n] = \sum_m h[m]\,h[n-m]$

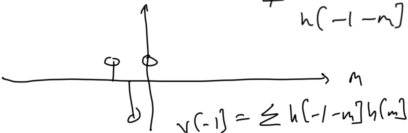




$y[-2] = \sum_n h[-2-m]\,h[m]$

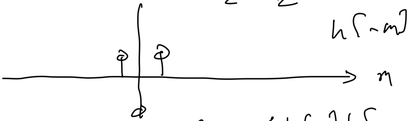
$= \frac{1}{4}$

h



$y[-1] = \sum_m h[-1-m]\,h[m]$

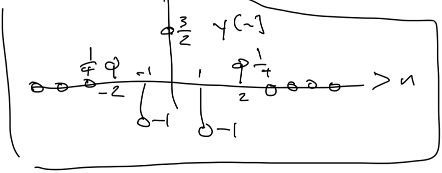
$= -\frac{1}{2} - \frac{1}{2} = -1$



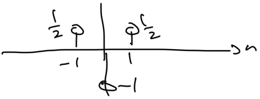
$y[0] = \sum_m h[m]\,h[-m]$

$= \frac{1}{4} + 1 + \frac{1}{4}$

$= \frac{6}{4} = \frac{3}{2}$



(3) $h[n] = -\delta[n] + \frac{1}{2}(\delta[n+1] + \delta[n-1])$



1a $H(\omega) = \sum_n h[n] e^{-j\omega n}$

$= \frac{1}{2} e^{j\omega} + \frac{1}{2} e^{-j\omega} - 1$

$= -1 + \cos(\omega)$

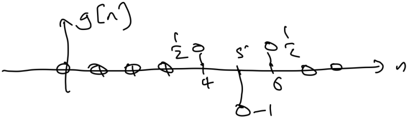
b $y[n] = h[n] * x[n]$

$z[n] = y[n-5] = y[n] * \delta[n-5]$

$z[n] = \left(\delta[n-5] * h[n]\right) * x[n]$

$g[n] = \delta[n-5] * h[n]$

$= h[n-5]$



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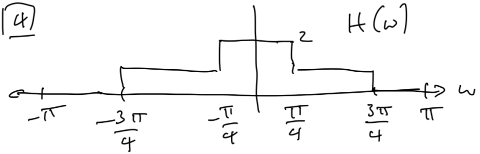


a $H(\omega) = LPF\left(\frac{3\pi}{4}\right) + LPF\left(\frac{\pi}{4}\right)$

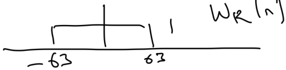
$h[n] = \left(\frac{\frac{3\pi}{4}}{\pi}\right) \text{sinc}\left(\frac{3\pi n}{4}\right) + \left(\frac{\pi/4}{\pi}\right) \text{sinc}\left(\frac{\pi n}{4}\right)$

$= \frac{3}{4} \text{sinc}\left(\frac{3\pi n}{4}\right) + \frac{1}{4} \text{sinc}\left(\frac{\pi n}{4}\right)$

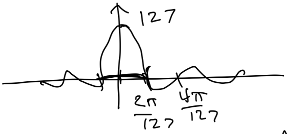
b HAMMING WINDOW : $\frac{4\pi}{N}$

TRANSITION BANDS ARE $\frac{4\pi}{127}$

Rectangular window



$$\longleftrightarrow \quad \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$



$$\frac{\omega N}{2} = \pi$$

$$\omega = \frac{2\pi}{N}$$

Hamming window

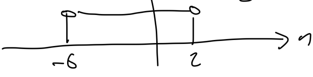
transition bandwidth $\approx 2 \cdot \frac{2\pi}{127}$

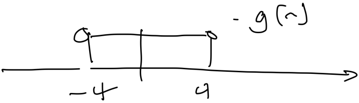
$$\boxed{= \frac{4\pi}{127}}$$

2021

$$h[n] = \begin{cases} 1 & -6 \le n \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$= g[n+2]$



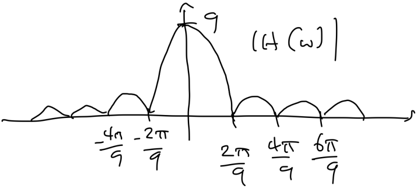


$$G(\omega) = \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

$$= \frac{\sin(9\omega/2)}{\sin(\omega/2)}$$

$$H(\omega) = e^{j2\omega}\frac{\sin(9\omega/2)}{\sin(\omega/2)}$$

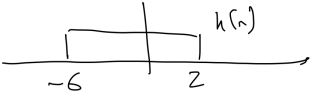
$$|H(\omega)| = \left|\frac{\sin(9\omega/2)}{\sin(\omega/2)}\right|$$

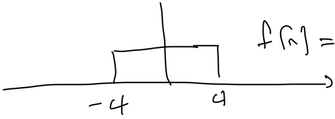


$$\frac{\sin(9\omega/2)}{\sin(\omega/2)} \approx \frac{\sin(9(\omega/2))}{\omega/2}$$

$$= 9\cdot\mathrm{sinc}\left(\frac{9\omega}{2}\right)$$

2)





$f[n] = g[n] * h[n]$

$$F(\omega) = \frac{\sin(9\omega/2)}{\sin(\omega/2)}$$

$$f[n] = h[n-2]$$

$$= h[n] * \delta[n-2]$$

$$g[n] = \delta[n-2]$$

3) $x(t) = \cos(2\pi 800 t)$

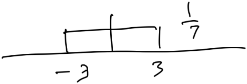
$F_s = 8000 \frac{\text{samples}}{\text{second}}$

$$x(n) = \cos\left(2\pi\, 800 \left(\frac{n}{8000}\right)\right)$$

$$= \cos(0.2\pi n)$$

$$y(n) = h(n) * x(n)$$

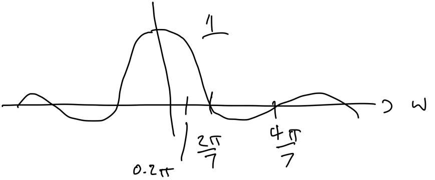
$h(n) =$



$$H(\omega) = \frac{1}{7} \cdot \frac{\sin(7\omega/2)}{\sin(\omega/2)}$$

$$y[n] = A \cos(0.2\pi n + \theta)$$

$$A = |H(0.2\pi)| = \left| \frac{\sin(0.2\pi\, 7/2)}{7 \sin(0.2\pi/2)} \right|$$

$$\theta = \measuredangle H(0.2\pi) = 0$$