Spring or Fall 2022

1) $x(t) = -12\cos(1000\pi t - \frac{\pi}{4}) + 4\sin(1000\pi t)$

$= M\cos(1000\pi t + \theta)$

Find x and y s.t. $M = \sqrt{x^2+y^2}$ and either $\theta = \mathrm{atan}(\frac{y}{x})$ or $\theta = \mathrm{atan}(\frac{y}{x}) - \pi$

$x(t) = \mathrm{Re}\{-12 e^{-j\frac{\pi}{4}} e^{j1000\pi t}\} + \mathrm{Re}\{-j4 e^{j1000\pi t}\}$

$z = \cos\theta + j\sin\theta = e^{j\theta}$

$-jz = -j\cos\theta - (j)^2\sin\theta$

$= -j\cos\theta + \sin\theta$

$x(t) = \mathrm{Re}\{e^{j1000\pi t}(-12e^{-j\frac{\pi}{4}} - j4)\}$

$M = |-4j - 12e^{-j\frac{\pi}{4}}|$

$\theta = \measuredangle(-4j - 12e^{-j\frac{\pi}{4}})$

$z = -4j - 12e^{-j\pi/4}$

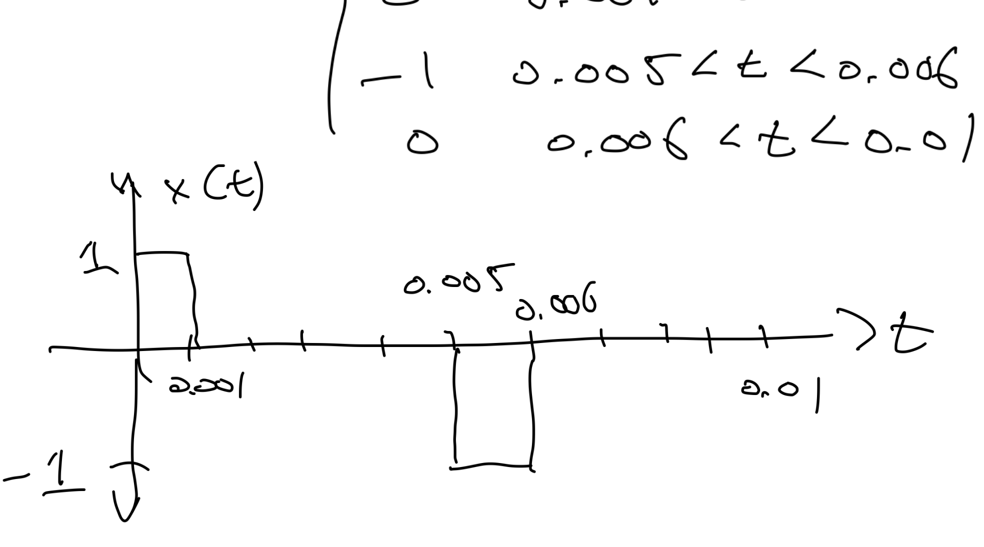
$= -4j - 12\cos(-\frac{\pi}{4}) - 12j\sin(-\frac{\pi}{4})$

$= -12\cos(-\frac{\pi}{4}) + j(-4 - 12\sin(-\frac{\pi}{4}))$

$x = -12\cos(-\frac{\pi}{4})$

$y = -4 - 12\sin(-\frac{\pi}{4})$

2) $x(t) = \begin{cases} 1 & 0 < t < 0.001 \\ 0 & 0.001 < t < 0.005 \\ -1 & 0.005 < t < 0.006 \\ 0 & 0.006 < t < 0.01 \end{cases}$



a) What is X_k for $k \neq 0$?

$X_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j\frac{2\pi k t}{T_0}} dt$

$= \frac{1}{0.01}\left[\int_0^{0.001} x(t) e^{-j\frac{2\pi k t}{0.01}} dt + \int_{0.005}^{0.006} x(t) e^{-j\frac{2\pi k t}{0.01}} dt\right]$

$= \frac{1}{0.01}\left(\left[\frac{1}{-j200\pi k} e^{-j200\pi k t}\right]_0^{0.001} - \left[\frac{1}{-j200\pi k} e^{-j200\pi k t}\right]_{0.005}^{0.006}\right)$

$= \frac{1}{0.01}\left(\frac{1}{-j200\pi k} e^{-j200\pi k(0.001)} - \frac{1}{-j200\pi k}\cdot 1 - \frac{1}{-j200\pi k} e^{-j200\pi k(0.006)} + \frac{1}{-j200\pi k} e^{-j200\pi k(0.005)}\right)$

b) $y(t) = \frac{dx}{dt}$

Y_k in terms of X_k

Formula sheet:

$y(t) = \frac{dx}{dt} = \sum_k (j2\pi f_k a_k) e^{j2\pi f_k t}$

$f_k = k F_0 = k\frac{1}{T} = 100k$

$y(t) = \sum_k j200\pi k X_k e^{j200\pi k t}$

$Y_k = j200\pi k X_k$

or, could answer

$Y_k = j2\pi k F_0 X_k$ for $F_0 = \frac{1}{0.01}$

3) $y[n] = x(\frac{n}{F_s})$ $t = \frac{n}{F_s}$

$F_s = 10{,}000$ samples/second

$y[n] \rightarrow$ D/A $\rightarrow z(t)$

$z(t) = \sum_{n=-\infty}^{\infty} y[n]\,\mathrm{sinc}\left(\pi\frac{(t - n/F_s)}{1/F_s}\right)$

a) $x(t) = 3\cos(2\pi 8000 t + \frac{\pi}{4})$

$\frac{F_s}{2} < f < F_s$:

$f_a = F_s - f$

$z_a = z^*$

$z(t) = 3\cos(2\pi 2000 t - \frac{\pi}{4})$

b) $x(t) = 3\cos(2\pi 12{,}000 t + \frac{\pi}{4})$

$F_s < f < \frac{3F_s}{2}$

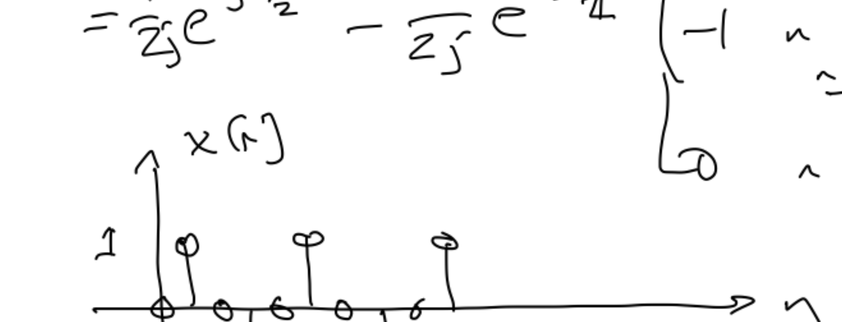
$f_a = f - F_s$

$z_a = z$

$z(t) = 3\cos(2\pi 2000 t + \frac{\pi}{4})$

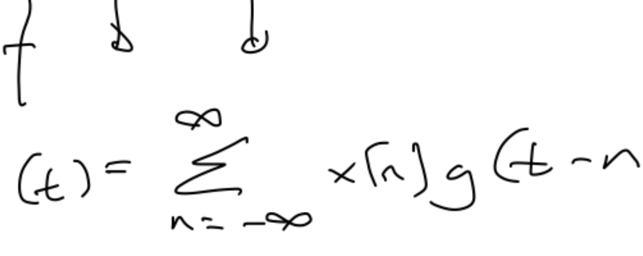
4) $x[n] = \sin(\frac{\pi n}{2}) = \begin{cases} 1 & n \text{ is odd}, \frac{n-1}{2} \text{ even} \\ -1 & n \text{ is odd}, \frac{n-1}{2} \text{ odd} \\ 0 & n \text{ is even} \end{cases}$

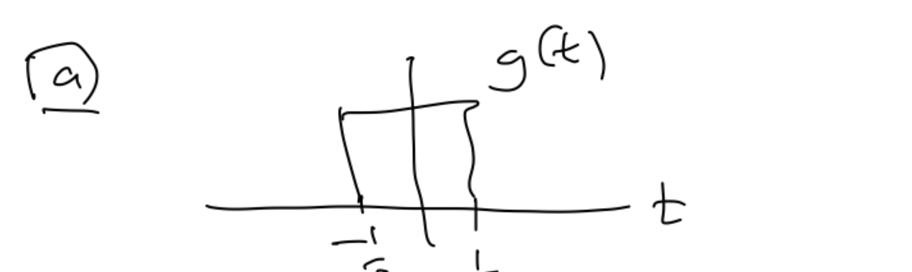
$= \frac{1}{2j}e^{j\frac{\pi n}{2}} - \frac{1}{2j}e^{-j\frac{\pi n}{2}}$



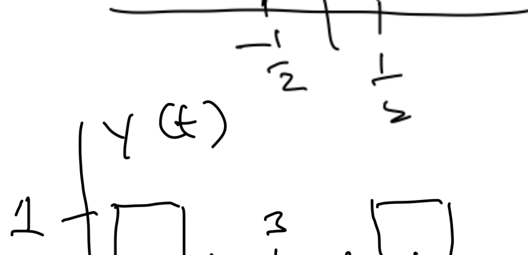
$y(t) = \sum_{n=-\infty}^{\infty} x[n]\, g(t-n)$

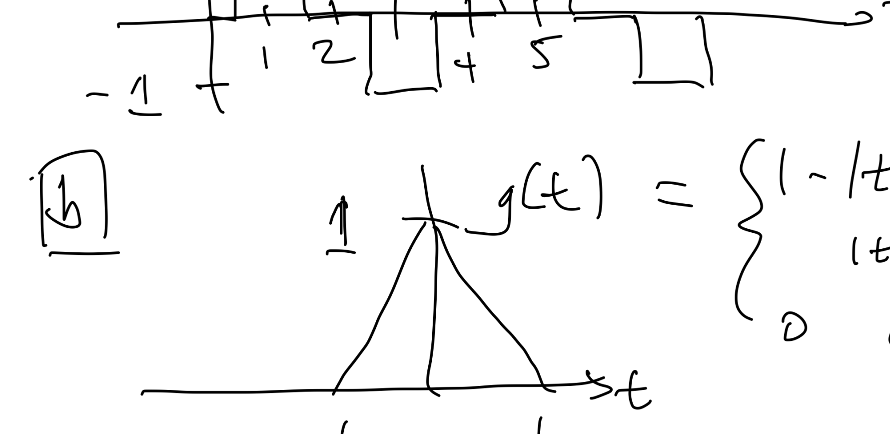
a)



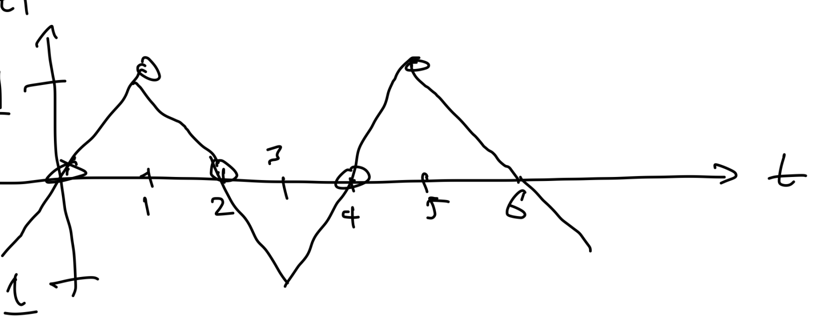


b) $g(t) = \begin{cases} 1 - |t| & |t| \le 1 \\ 0 & \text{else} \end{cases}$





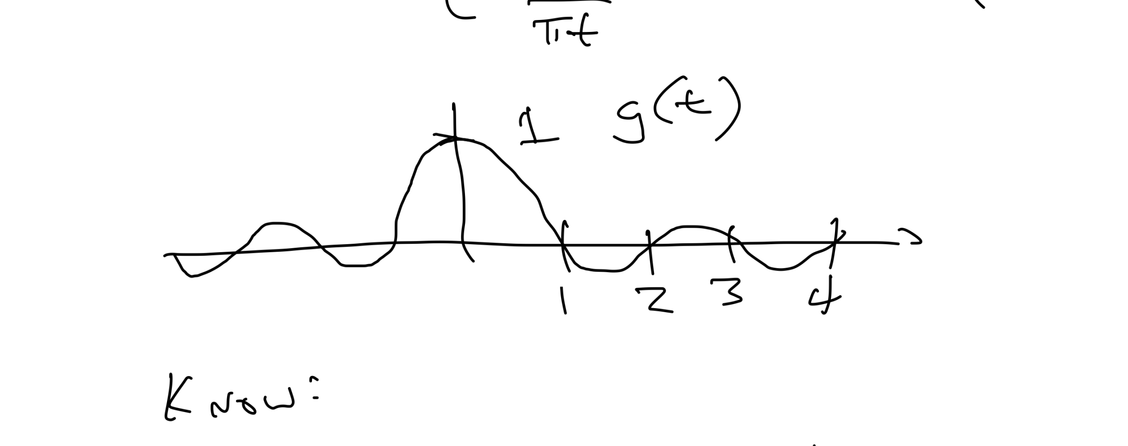
c) $g(t) = \begin{cases} 1 & t = 0 \\ \frac{\sin(\pi t)}{\pi t} & \text{otherwise} \end{cases}$

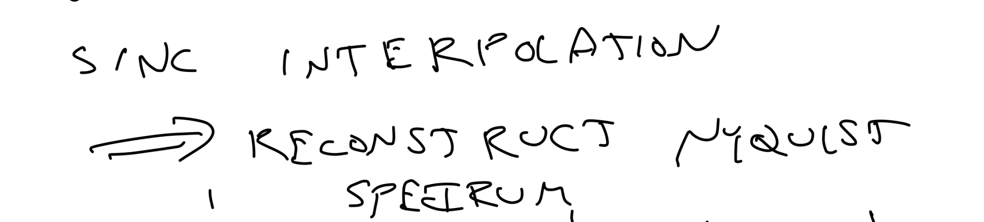


Know:

Sinc interpolation

$\Rightarrow$ Reconstruct Nyquist spectrum





$\frac{1}{2j}e^{j2\pi\frac{F_s}{2}t} - \frac{1}{2j}e^{-j2\pi\frac{F_s}{2}t}$

$= \frac{1}{2j}(e^{j2\pi f t} - e^{-j2\pi f t})$

$= \frac{1}{2j}2j\sin(2\pi f t)$

$= \sin(2\pi f t)$ $f = \frac{F_s}{2} = \frac{1}{2}$

$y(t) = \sin(\pi t)$