

# ECE 401 Signal and Image Analysis

## Homework 4

UNIVERSITY OF ILLINOIS  
Department of Electrical and Computer Engineering

Assigned: 10/9/2023; Due: 10/18/2023  
Reading: *DSP First* Chapter 6

### Problem 4.1

Consider this filter:

$$y[n] = x[n] + x[n - 1]$$

Show that the magnitude response of this filter is  $|H(\omega)| = 2 \cos(\omega/2)$ .

**Solution:**

$$h[n] = \begin{cases} 1 & n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

$$H(\omega) = \sum_n h[n]e^{-j\omega n} = 1 + e^{-j\omega} = e^{-j\omega/2}(e^{j\omega/2} + e^{-j\omega/2}) = e^{-j\omega/2}2 \cos(\omega/2)$$

### Problem 4.2

Suppose you have a filter whose frequency response is

$$H(\omega) = 14e^{-j6\omega}$$

Show that, if  $x[n] = \cos(\omega n)$ , the effect of convolving  $y[n] = x[n] * h[n]$  is to

- (a) scale  $x[n]$  by a factor of 14, and
- (b) delay it by 6 samples.

**Solution:** Using the frequency response formula,

$$\begin{aligned} y[n] &= |H(\omega)| \cos(\omega n + \angle H(\omega)) \\ &= 14 \cos(\omega n - 6\omega) \\ &= 14 \cos(\omega(n - 6)) \end{aligned}$$

### Problem 4.3

The signals  $x_1(t)$  and  $x_2(t)$  are cosines an octave apart (roughly C6 and C7):

$$\begin{aligned}x_1(t) &= \cos(2\pi 1000t) \\x_2(t) &= \cos(2\pi 2000t)\end{aligned}$$

The signals are sampled (at  $F_s = 16000$  samples/second), then the resulting signals  $x_1[n]$  and  $x_2[n]$  are passed through a first-difference operator:

$$\begin{aligned}y_1[n] &= x_1[n] - x_1[n-1] \\y_2[n] &= x_2[n] - x_2[n-1]\end{aligned}$$

What are the amplitudes of the signals  $y_1[n]$  and  $y_2[n]$ ?

**Solution:** In radians/second, the frequencies are

$$\omega_1 = \frac{2\pi 1000}{16000} = \frac{\pi}{8}, \quad \omega_2 = \frac{2\pi 2000}{16000} = \frac{\pi}{4}$$

The frequency response of the first-difference operator is

$$H(\omega) = 1 - e^{-j\omega} = 2je^{-j\omega/2} \sin(\omega/2)$$

Which has these magnitude responses:

$$\left| H\left(\frac{\pi}{8}\right) \right| = 2 \sin(\pi/16) \approx 0.39, \quad \left| H\left(\frac{\pi}{4}\right) \right| = 2 \sin(\pi/8) \approx 0.77$$

#### Problem 4.4

The signals  $x_1(t)$  and  $x_2(t)$  are cosines an octave apart (roughly C6 and C7):

$$\begin{aligned}x_1(t) &= \cos(2\pi 1000t) \\x_2(t) &= \cos(2\pi 2000t)\end{aligned}$$

The signals are sampled (at  $F_s = 16000$  samples/second), then the resulting signals  $x_1[n]$  and  $x_2[n]$  are passed through a seven-sample local average:

$$\begin{aligned}y_1[n] &= \frac{1}{7} \sum_{m=-3}^3 x_1[n-m] \\y_2[n] &= \frac{1}{7} \sum_{m=-3}^3 x_2[n-m]\end{aligned}$$

What are the amplitudes of the signals  $y_1[n]$  and  $y_2[n]$ ?

**Solution:** In radians/second, the frequencies are

$$\omega_1 = \frac{2\pi 1000}{16000} = \frac{\pi}{8}, \quad \omega_2 = \frac{2\pi 2000}{16000} = \frac{\pi}{4}$$

The frequency response of the seven-sample central local average filter is

$$H(\omega) = \frac{\sin(7\omega/2)}{7 \sin(\omega/2)}$$

Which has these magnitude responses:

$$\left| H\left(\frac{\pi}{8}\right) \right| = \frac{\sin(7\pi/16)}{7 \sin(\pi/16)} \approx 0.72, \quad \left| H\left(\frac{\pi}{4}\right) \right| = \frac{\sin(7\pi/8)}{7 \sin(\pi/8)} \approx 0.14$$