ECE 401 Signal and Image Analysis
Homework 2

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: 9/4/2023; Due: 9/13/2023
Reading: *DSP First* pp. 12-34, 50-58, 61-71

Problem 2.1

Suppose that

$$v(t) = 2\cos(2\pi 880t) + 2\sin(2\pi 1320t)$$

What is the fundamental frequency? What are the Fourier series coefficients, V_k?

Problem 2.2

Suppose that $x(t)$ is a square wave with a period of T_0, and with the following definition:

$$x(t) = \begin{cases} \frac{1}{2} & -\frac{T_0}{4} < t < \frac{T_0}{4} \\ -\frac{1}{2} & \frac{T_0}{4} < t < \frac{3T_0}{4} \end{cases}$$

We showed in class that the Fourier coefficients of this square wave are

$$X_k = \begin{cases} 0 & k \text{ even} \\ \frac{(-1)^{(|k|-1)/2}}{\pi k} & k \text{ odd} \end{cases}$$

Remember that the real-valued coefficients correspond to a Fourier series where all of the components are cosines. That makes sense, because the signal has even symmetry ($x(t) = x(-t)$), just like a cosine.

Suppose that we delay the signal by one quarter period, to produce the signal

$$y(t) = \begin{cases} \frac{1}{2} & 0 < t < \frac{T_0}{2} \\ -\frac{1}{2} & \frac{T_0}{2} < t < T_0 \end{cases}$$

Notice that $y(t)$ has odd symmetry ($y(t) = -y(-t)$), just like a sine wave. In that case, we might speculate that the Fourier seires expansion will be composed entirely of sine waves, i.e., the Fourier series coefficients, Y_k, will all be imaginary numbers.

Use the time-delay property of the spectrum (from lecture 3) to find out what happens to X_k when $x(t)$ is delayed by exactly one quarter period.

Problem 2.3

Suppose that $z(t)$ is a triangle wave with a period of T_0, and with the following definition:

$$z(t) = \begin{cases} \frac{t}{2} - \frac{T_0}{4} & 0 < t < \frac{T_0}{2} \\ -\frac{t}{2} + \frac{3T_0}{4} & \frac{T_0}{2} < t < T_0 \end{cases}$$

Notice that this signal is exactly the anti-derivative of the signal $y(t)$ from problem (1), i.e., $y(t) = dz/dt$. Use the differentiation property of the spectrum (from lecture 3) in order to find the Fourier series coefficients Z_k.

Problem 2.4

Suppose that a violin is playing the note A4 (440Hz), but our recording quality is bad, so we only get the first two harmonics:

$$x(t) = \sum_{k=-2}^{2} a_k e^{j2\pi k440t}$$

Suppose we measure the spectrum, and find it to be

$$\{(-880, 0.01), (-440, 1), (0, 0), (440, 1), (880, 0.01)\}$$

In order to improve the balance a little, we try differentiating the tone. Our differentiator also imposes a delay and a DC offset, though, so what we get is

$$y(t) = \frac{dx(t - 0.001)}{dt} + 1.5$$

Find the spectrum of $y(t)$.