ECE 401 Signal and Image Analysis
Homework 1

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: 8/21/2023; Due: 8/30/2023
Reading: *DSP First* pp. 12-34, 50-58, 61-71

Problem 1.1

Find $\angle z$ as a function of a and b.

$$z = e^{ja} + e^{jb} \tag{1.1-1}$$

Problem 1.2

In standard tuning, the middle A note on a piano (A4) has a frequency of 440Hz. Consider the note

$$x(t) = 14\cos\left(2\pi 440t + 0.88\pi\right)$$

Sketch one complete period of $x(t)$, from its first peak after $t = 0$ until its second peak after $t = 0$. Label the times of both peaks, and the value of $x(t)$ at both peaks.

Problem 1.3

Suppose you're given the signal

$$x(t) = \cos(2\pi 440t) + 3\sin(2\pi 440t)$$

Find the phasor representation of $x(t)$, and simplify it to polar form. You might want to take advantage of facts like $\sin(x) = \cos(x - \frac{\pi}{2})$, and $\sin(\frac{\pi}{2}) = 1$, and $\cos(\frac{\pi}{2}) = 0$.

Problem 1.4

Kwikwag's beat-tones example on Wikipedia adds two tones, at the frequencies 110Hz and 104Hz:

$$x(t) = \cos(2\pi 110t) + \cos(2\pi 104t)$$

Find a sequence of frequencies and phasors, $\{(f_{-2}, a_{-2}), \ldots, (f_2, a_2)\}$, such that

$$x(t) = \sum_{k=-2}^{2} a_k e^{j2\pi f_k t}$$