

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 401 SIGNAL AND IMAGE ANALYSIS
Spring 2022

EXAM 2

Monday, October 31, 2022

- This is a **CLOSED BOOK** exam.
- You are permitted one sheet of handwritten notes, 8.5x11.
- Calculators and computers are not permitted.
- Do not simplify explicit numerical expressions. The expression “ $e^{-5} \cos(3)$ ” is a MUCH better answer than “-0.00667”.
- If you’re taking the exam online, you will need to have your webcam turned on. Your exam will appear on Gradescope at exactly 1:00pm; you will need to photograph and upload your answers by exactly 2:00pm.
- There will be a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must **SHOW YOUR WORK** to get full credit.

Name: _____

Convolution

$$h[n] * x[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

Frequency Response

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$
$$h[n] * \cos(\omega n) = |H(\omega)| \cos(\omega n + \angle H(\omega))$$

Rectangular Window and Ideal LPF

$$w_R[n] = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \leftrightarrow W_R(\omega) = e^{-j\omega(N-1)/2} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$
$$h_{\text{ideal}}[n] = \frac{\omega_c}{\pi} \text{sinc}(\omega_c n) \leftrightarrow H_{\text{ideal}}(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Hamming Window

$$w_H[n] = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$
$$W_H(\omega) = 0.54W_R(\omega) + 0.23W_R\left(\omega - \frac{2\pi}{N-1}\right) + 0.23W_R\left(\omega + \frac{2\pi}{N-1}\right)$$

1. (25 points) Consider the following system:

$$y[n] = \begin{cases} x[n] & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

(a) Is this system linear? Prove your answer.

(b) Is this system shift-invariant? Prove your answer.

2. (25 points) The second-derivative of a sampled signal may be approximated by an LSI system with the following impulse response:

$$h[n] = -\delta[n] + \frac{1}{2} (\delta[n+1] + \delta[n-1])$$

(a) Is this system causal? Why or why not?

(b) Is this system stable? Why or why not?

(c) What is $h[n] * h[n]$? Give the time and value of every nonzero sample, using either a sketch or an equation.

3. (25 points) Consider, again, the same LSI system that was used in problem 2. Remember that its impulse response is

$$h[n] = -\delta[n] + \frac{1}{2} (\delta[n+1] + \delta[n-1])$$

- (a) What is its frequency response?

- (b) Consider a system \mathcal{G} that accepts as input a signal $x[n]$, and generates as output a signal $z[n]$. The system \mathcal{G} does two things to $x[n]$. First, it convolves it with $h[n]$, producing $y[n] = x[n] * h[n]$. Second, it delays it by five samples, producing $z[n] = y[n-5]$. What is the impulse response of the system \mathcal{G} ?

4. (25 points) You have an application for which it is necessary to increase the amplitude of inputs below $\frac{\pi}{4}$ radians/sample, while eliminating inputs above $\frac{3\pi}{4}$ radians/sample. The ideal filter is therefore:

$$H_i(\omega) = \begin{cases} 2 & |\omega| < \frac{\pi}{4} \\ 1 & \frac{\pi}{4} < |\omega| < \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} < |\omega| < \pi \end{cases}$$

- (a) What is $h_i[n]$?

- (b) You propose to create a realizable filter, $h[n]$, by windowing $h_i[n]$ with a length-127 Hamming window: $h[n] = w[n]h_i[n]$. In the resulting frequency response, $H(\omega)$, how wide are the transition bands?

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