UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 401 Signal and Image Analyais
Spring 2021

EXAM 2

Monday, November 1, 2021

- This is a CLOSED BOOK exam.
- You are permitted one sheet of handwritten notes, 8.5x11.
- Calculators and computers are not permitted.
- Do not simplify explicit numerical expressions. The expression "$e^{-5}\cos(3)$" is a MUCH better answer than "-0.00667".
- If you're taking the exam online, you will need to have your webcam turned on. Your exam will appear on Gradescope at exactly 1:00pm; you will need to photograph and upload your answers by exactly 2:00pm.
- There will be a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Name: ______________________________

Convolution

$$h[n] * x[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

Frequency Response

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

$$h[n] * \cos(\omega n) = |H(\omega)| \cos\left(\omega n + \angle H(\omega)\right)$$

Rectangular Window and Ideal LPF

$$w_R[n] = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \leftrightarrow W_R(\omega) = e^{-\frac{j\omega(N-1)}{2}} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

$$h_{\text{ideal}}[n] = \frac{\omega_c}{\pi} \text{sinc}(\omega_c n) \leftrightarrow H_{\text{ideal}}(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Hamming Window

$$w_H[n] = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

1. (20 points) Consider the following filter:

$$h[n] = \begin{cases} 1 & -6 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Sketch $|H(\omega)|$ as a function of ω, for $-\pi < \omega < \pi$. Label the amplitude at $\omega = 0$, and at least two of the frequencies at which $|H(\omega)| = 0$.

Solution:

$$|H(\omega)| = \left| \frac{\sin(9\omega/2)}{\sin(\omega/2)} \right|$$

The amplitude at $\omega = 0$ is 9. $|H(\omega)| = 0$ at the frequencies $\omega = \frac{2\pi k}{9}$, for any integer k that is not a multiple of 9.

2. (20 points) Consider the following filter:

$$h[n] = \begin{cases} 1 & -6 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Design a filter $g[n]$ such that $F(\omega)$ is real valued (i.e., its imaginary part is zero, $\Im\{F(\omega)\} = 0$), where

$$f[n] = g[n] * h[n]$$

$$g[n] =$$

Solution:

$$g[n] = \delta[n-2]$$

3. (20 points) The signal $x(t) = \cos(2\pi 800 t)$ is sampled at $F_s = 8000$ samples/second, then processed by the filter

$$y[n] = \frac{1}{7} \sum_{m=-3}^{3} x[n-m]$$

The resulting signal is $y[n] = A\cos(\omega n + \theta)$. What are A, ω, and θ?

$A =$

Solution:

$$A = \frac{\sin(2\pi 800 \times 7/(2 \times 8000))}{7\sin(2\pi 800/(2 \times 8000))}$$
$$= \frac{\sin(7\pi/10)}{7\sin(\pi/10)}$$

$\omega =$

Solution:

$$\omega = \frac{2\pi 800}{8000} = \frac{\pi}{5}$$

$\theta =$

Solution:

$$\theta = 0$$

4. (20 points) Consider the system

$$y[n] = \frac{1}{4}x[n+2] + \frac{1}{2}x[n+1] + x[n] - x[n-1] - \frac{1}{2}x[n-2] - \frac{1}{4}x[n-3]$$

Sketch the impulse response of this system as a function of n, for $-5 \leq n \leq 5$. Clearly label the values of every nonzero sample.

Solution: The non-zero samples are

$$h[n] = \begin{cases} \frac{1}{4} & n = -2 \\ \frac{1}{2} & n = -1 \\ 1 & n = 0 \\ -1 & n = 1 \\ -\frac{1}{2} & n = 2 \\ -\frac{1}{4} & n = 3 \end{cases}$$

5. (20 points) You want to approximate the following ideal bandpass filter:

$$H_{\text{ideal}}(\omega) = \begin{cases} 1 & 0.3\pi < |\omega| < 0.4\pi \\ 0 & \text{otherwise} \end{cases}$$

Design an FIR filter $h[n]$ that is exactly 64 samples long, such that $|H(\omega)| \approx |H_{\text{ideal}}(\omega)|$. Window $h[n]$ in such a way that the first sidelobe has a level of less than -40dB relative to the mainlobe.

$$h[n] =$$

Solution:

$$h[n] = w_H[n]\left(0.4\text{sinc}(0.4\pi(n-31.5)) - 0.3\text{sinc}(0.3\pi(n-31.5))\right)$$

where

$$w_H[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{63}\right) & 0 \le n \le 63 \\ 0 & \text{otherwise} \end{cases}$$