UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Principles of Signal Analysis
Fall 2014

# EXAM 3 SOLUTIONS

Friday, DECEMBER 19, 2014

- This is a CLOSED BOOK exam. You are allowed one page, front and back, of hand-written notes.
- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

| Problem | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| Total | |

**Name:** ______________________________________________

**Problem 1 (25 points)**

$x[n]$ is a white noise signal with power $\sigma_x^2 = 25$. $y[n]$ is a filtered signal, computed as:

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

Find $R_{yy}[\tau]$, the autocorrelation of $y[n]$.

**Problem 2 (25 points)**

Real-world signals (real speech and music signals, real images, etc) often have a $1/f$ frequency dependence, meaning that the signal $s[n]$ has the power spectrum

$$S_{ss}(\omega) = \begin{cases} C/B & |\omega| \le B \\ C/|\omega| & B \le |\omega| \le \pi \end{cases}$$

where $B$ and $C$ are some constants that depend on the type of signal.

Suppose that $x[n] = s[n] + v[n]$, where $v[n]$ is a white noise signal, uncorrelated with $s[n]$, that has a power of $\sigma_v^2$.

Suppose that you want to compute $y[n] = h[n] * x[n]$ in order to minimize the mean-squared error $E[(y[n] - s[n])^2]$.

Find the frequency response $H(\omega)$ of the Wiener filter. Your answer should be a function of the constants $B$, $C$, and $\sigma_v^2$.

**Problem 3 (25 points)**

In order to become a billionaire, you've decided you need to know what was the total value of the U.S. GDP every day of every year since 1901. Unfortunately, GDP figures are only published once per year (once per 365 days), so you need to interpolate them.

Consider the following system:

$$d[n] = \sum_{m=-\infty}^{\infty} y[m]g[n-365m] \tag{1}$$

where $y[m]$ is the GDP in the $m^{\text{th}}$ year, and $d[n]$ is the estimated GDP in the $n^{\text{th}}$ day.

Design the filter $g[n]$ so that Eq. 1 implements **PIECE-WISE LINEAR** interpolation. (Draw a sketch of $g[n]$ that specifies the values of all of its samples, or write a formula that does so).

**Solutions:**

$$g[n] = \begin{cases} 1 - \frac{|n|}{365} & -365 \leq n \leq 365 \\ 0 & \text{otherwise} \end{cases}$$

**Problem 4 (25 points)**

You have a signal $s[n]$ sampled at exactly 44,000 samples/second, but the high-frequency part of $s[n]$ (the part above 8kHz) is useless to you. You decide to save disk space by downsampling to create a signal $z[n]$, sampled at 16,000 samples/second. You decide to use the following strategy:

$$s[n] \longrightarrow \boxed{\uparrow 4} \longrightarrow \boxed{g[n]} \longrightarrow \boxed{\downarrow 11} \longrightarrow z[n]$$

In words: you upsample by a factor of 4, filter by $g[n]$, and downsample by a factor of 11. Design the filter $g[n]$ so that there is no aliasing. (Draw a sketch of $g[n]$ that specifies the values of all of its samples, or write a formula that does so).