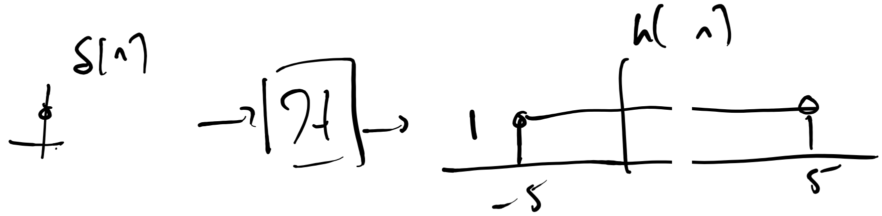
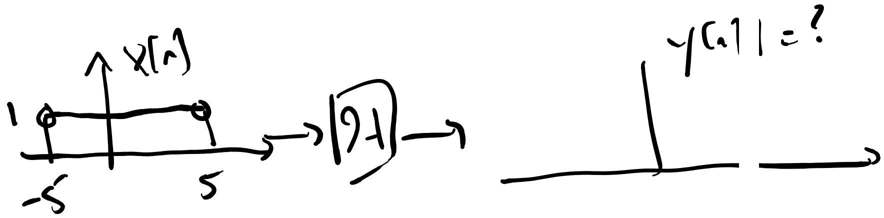
2022oct09

Wednesday, October 12, 2022 13:30



NON-CAUSAL B/c h[n] NOT RIGHT-SIDED

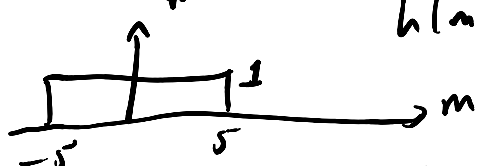
STABLE B/c $\sum_{n=-\infty}^{\infty} |h[n]|$ FINITE



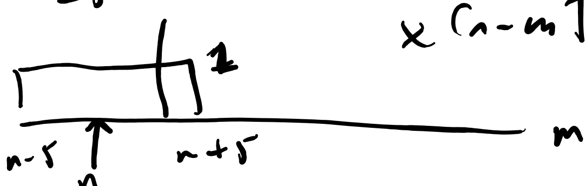
① Solve by convolution

$$y[n] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

h[m] ←



x[n-m] ←



h[m] x[n-m]

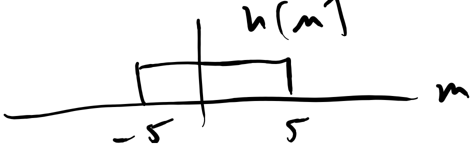
$$y[n] = \sum_m h[m]\, x[n-m]$$

$$= \sum_{m=-5}^{n+5} 1 \qquad = \sum_{m=0}^{n+10} 1$$

$$= n + 11 \qquad \text{FOR} \quad \begin{matrix} n+5 \geq -5 \\ n+5 \leq 5 \end{matrix}$$

$$\text{FOR} \quad \begin{matrix} n \geq -10 \\ n \leq 0 \end{matrix}$$

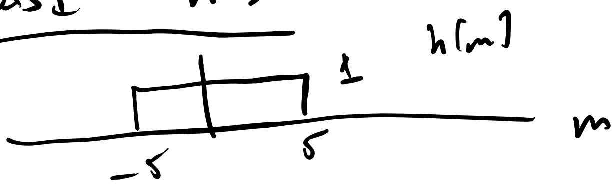
CASE $n < -10$:

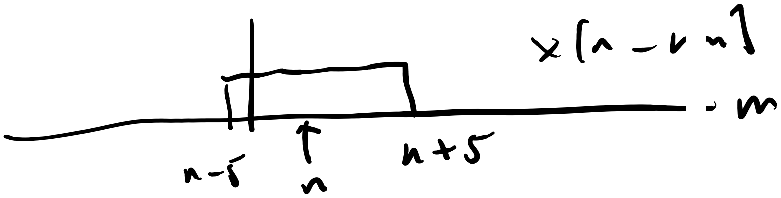


$x[n-m]h[m] = 0$ everywhere

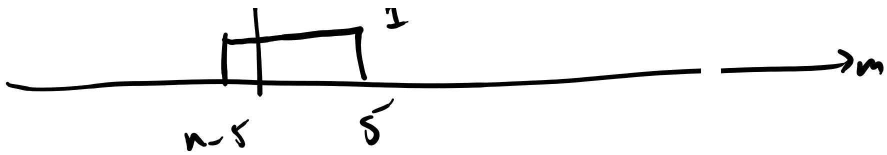
so $y[n] = \sum_m x[n-m]h[m] = 0$

CASE $n > 0$:





$x[n-m]h[m]$



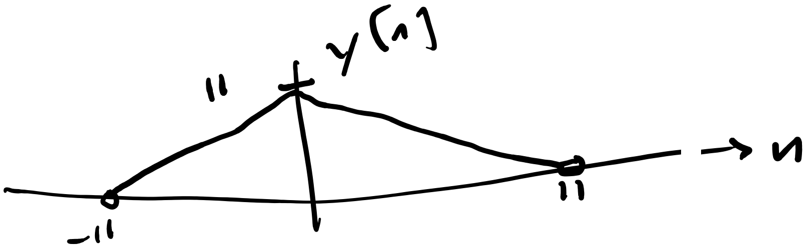
$$y[n] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

$$= \sum_{m=n-5}^{5} 1 = \sum_{m=n-10}^{0} 1$$

$$= 0 - (n-10) + 1$$

$$= 11 - n$$

$$y[n] = \begin{cases} 0 & n < -10 \\ 11+n & -10 \le n \le 0 \\ 11-n & 0 \le n \le 10 \\ 0 & n \ge 10 \end{cases}$$



(2) solve $Y(\omega) = H(\omega)X(\omega)$

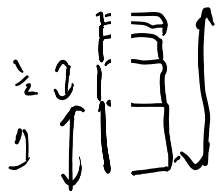
$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-5}^{5} e^{-j\omega n}$$

$$m = n + 5$$

$$= \sum_{m=0}^{10} e^{-j\omega(m-5)}$$

$$= e^{j\omega 5} \sum_{m=0}^{10} e^{-j\omega m} = e^{j\omega 5} \frac{1 - e^{-j\omega 11}}{1 - e^{-j\omega}}$$



$$\sum_{m=0}^{\infty} \left(\frac{1}{2}\right)^m = \frac{1}{1-\frac{1}{2}} = 2$$

USEFUL FACT

$$\sum_{m=0}^{\infty} a^m = \frac{1}{1-a}$$

$$\sum_{m=0}^{10} a^m = \sum_{m=0}^{\infty} a^m - \sum_{m=11}^{\infty} a^m$$

$$= \frac{1}{1-a} - a^{11} \sum_{m=0}^{\infty} a^{m}$$

$$= \frac{1}{1-a} - a^{11} \frac{1}{1-a}$$

$$= \frac{1 - a^{11}}{1-a}$$

N

$$\sum_{m=0}^{N-} a^m = \frac{1-a}{1-a}$$

$$X(\omega) = e^{+5j\omega}\,\frac{1-e^{-j11\omega}}{1-e^{-j\omega}} = H(\omega)$$

$$Y(\omega) = H(\omega)\,X(\omega)$$

$$= e^{+10j\omega}\left(\frac{1-e^{-11j\omega}}{1-e^{-j\omega}}\right)^2$$