UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 401 Signal and Image Analyais
Spring 2021

PRACTICE EXAM 3

Exam 3 will be held Tuesday, December 14, 8:00-11:00am

- This will a CLOSED BOOK exam.
- You will be permitted two sheets of handwritten notes, 8.5x11.
- Calculators and computers will not be permitted.
- Do not simplify explicit numerical expressions. The expression "$e^{-5}\cos(3)$" is a MUCH better answer than "-0.00667".
- If you're taking the exam online, you will need to have your webcam turned on. Your exam will be sent to you by e-mail and on zoom at exactly 8:00am; you will need to photograph and upload your answers by exactly 11:00am.
- There will be a total of 200 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Name: ______________________________

1. (20 points) Consider the following signal:

$$x[n] = \delta[n-15] + \delta[n-30]$$

(a) $X[k]$ is the 32-point DFT of $x[n]$. Specify $X[k]$ as a function of k.

(b) Suppose that $h[n]$ is defined as follows:

$$h[n] = \begin{cases} e^{-n/14} & 0 \le n \le 14 \\ 0 & \text{otherwise} \end{cases}$$

Suppose that $H[k]$ is the 32-point DFT of $h[n]$, $Y[k] = H[k]X[k]$, and $y[n]$ is the inverse DFT of $Y[k]$. Find $y[n]$.

2. (20 points) Consider the following system. The input of this system is $x[n]$, and the output is $y[n]$:

$$v[n] = x[n] + 0.9v[n-1]$$
$$y[n] = v[n] - 0.7y[n-1]$$

(a) What is the system function, $H(z)$, for this system?

(b) What is the impulse response of this system?

3. (20 points) A flute player is playing a middle-A note. This system can be well modeled by blowing white noise through a damped resonator with a resonant frequency of 440Hz, and a bandwidth of 20Hz. Suppose you want to synthesize this flute digitally, by blowing white noise through a second-order damped resonator, with a sampling frequency of $F_s = 10,000$Hz.

   (a) You want to implement the resonator as

$$y[n] = x[n] + a_1 y[n-1] + a_2 y[n-2]$$

   What are a_1 and a_2?

(b) Suppose you have succeeded in implementing the digital filter. Now you give it the input $x[n] = \delta[n]$. What is the output?

4. (20 points) You have recorded an electrocardiogram signal, $x[n]$, with a sampling frequency of $F_s = 1.2kHz$. Unfortunately, it has been corrupted by power line noise: it has a big sinusoidal component at 60Hz. Fortunately, you know how to eliminate power line noise using a notch filter. All you have to do is to pass the signal through a difference equation:

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2] - a_1 y[n-1] - a_2 y[n-2] \tag{1}$$

Use a pole amplitude of 0.98. What are b_1, b_2, a_1, and a_2?

(Note: leave your answer in the form of an explicit numerical expression. For example, if you discover that $a_1 = (0.3)^2 \sin(2\pi/400)$, then you should leave it in that form instead of trying to simplify.)

5. (10 points) Consider the signal $x(t) = -2 + \sin(40\pi t)$. Suppose we sample this signal at $F_s = 100$Hz, then take a length-20 DFT of 20 samples of this signal. For which values of k, $0 \leq k \leq 19$, will the DFT samples $X[k]$ be nonzero?

6. (20 points) Determine whether the following LTI system is causal and/or BIBO stable:

$$y[n] = x[n+1] + y[n-1]$$

(a) Is it causal? Describe your reasoning in words.

(b) Is it stable? Prove your answer.

7. (20 points) A particular system generates an output $y[n]$ from its input $x[n]$ according to the following rule:

$$y[n] = \begin{cases} x[n] & n \text{ is even} \\ \frac{1}{2}\left(x[n-1] + x[n+1]\right) & n \text{ is odd} \end{cases}$$

(a) Is the system causal? Give your reason.

(b) Is the system stable? Give your reason.

8. (15 points) A signal is corrupted with narrowband noise at $\omega_n = \frac{\pi}{6}$ radians/sample. To remove the noise, you create a notch filter with a pole magnitude of $r = 0.9$. The filter system function is

$$H(z) = \frac{(1 - r_1 z^{-1})(1 - r_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$$

Speciy the poles and zeros in magnitude/phase form (e.g., $p_1 = me^{j\theta}$, but you should specify the numerical value of m and the numerical value of θ).

9. (15 points) Suppose you want to implement a filter with the following frequency response:

$$H(\omega) = \frac{(1 - 0.5e^{j\pi/4}e^{-j\omega})(1 - 0.5e^{-j\pi/4}e^{-j\omega})}{(1 + 0.5e^{j\pi/4}e^{-j\omega})(1 + 0.5e^{-j\pi/4}e^{-j\omega})}$$

You realize that you can get this frequency response by writing one line of code in matlab, implementing the following equation:

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2] + a_1 y[n-1] + a_2 y[n-2]$$

Find the filter coefficients.

10. (30 points) Suppose you have a signal sampled at $F_s = 600$ samples/second. You wish to create a notch filter to eliminate a noise component at 60Hz. You choose to do this using the following filter:

$$H(z) = \frac{(1 - r_1 z^{-1})(1 - r_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$$

(a) Specify the values of the poles p_1, p_2 and the zeros r_1, r_2. Note that there is one free parameter in your answer that is not specified by the problem statement; you may set that free parameter to any reasonable value.

(b) Now suppose you are given a system function

$$H(z) = \frac{(1 - r_1 z^{-1})(1 - r_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$$

and you wish to implement this using the equation

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2] + a_1 y[n-1] + a_2 y[n-2]$$

Find b_1, b_2, a_1 and a_2 in terms of r_1, r_2, p_1 and p_2.