UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498DLJ Principles of Signal Analysis
Fall 2011

FINAL EXAM

Thursday, December 15, 2011, 8:00-11:00

- This is a CLOSED BOOK exam.
- You are allowed one page (front and back) of hand-written notes. Calculators are not permitted.
- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score	Problem	Score	Total
1		6		
2		7		
3		8		
4		9		
5		10		
Total		Total		

Name: ____

Useful Formulas

Angle	Cosine	Sine
0	1	0
$\frac{\pi}{2}$	0	1
π	-1	0

Continuous Time Fourier Transform (CTFT)

$$X_c(\Omega) = \int_{-\infty}^{\infty} x_c(t) e^{-j\Omega t} dt$$

$$x_c(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_c(\Omega) e^{j\Omega t} d\Omega$$

Discrete Time Fourier Transform (DTFT)

$$X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

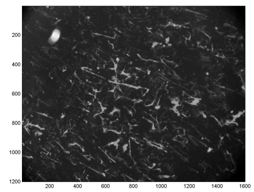
$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_d(\omega) e^{j\omega n} d\omega$$

Discrete Fourier Transform (DFT)

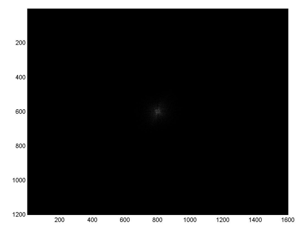
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi kn}{N}}$$

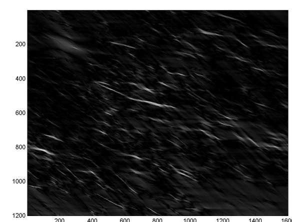
Problem 1 (10 points)



The image shown above at left has been subjected to several different types of image transforms. Draw a line between the name of the image transform (left column) and the corresponding transformed image (right column).

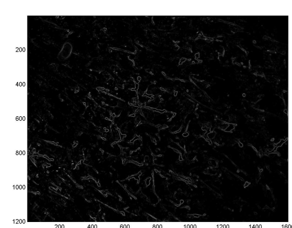


(a) Histogram Equalized

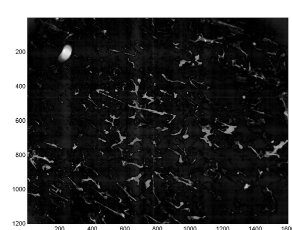


(b) Sobel Filtered

(c) Fourier Transform



(d) Filtered to Remove DC using a Frequency-Sampling Filter



(e) Wedge Filtered

Problem 2 (10 points)

Many types of audio signal processing begin with a step called "pre-emphasis," usually defined by the equation

$$y[n] = x[n] - 0.95x[n-1]$$

Suppose that the input is $x[n] = x_c(nT)$, where $x_c(t)$ is a bandlimited signal such that $X_c(\Omega) = 0$ for $|\Omega| > \frac{\pi}{T}$. Suppose also that the output is converted back into continuous time according to

$$y_c(t) = \sum_{n=-\infty}^{\infty} y[n]\mathrm{sinc}(t - nT)$$

where $1/T = 10,000$ samples/second. Find the frequency response $H(\Omega) = Y(\Omega)/X(\Omega)$ of the equivalent continuous-time system. Sketch the magnitude response $|H(\Omega)|$ for $-\frac{\pi}{T} \leq \Omega \leq \frac{\pi}{T}$. Label the values $|H(0)|$, $|H(\frac{\pi}{2T})|$, and $|H(\frac{\pi}{T})|$. You may find the table of cosines and sines on the "Useful Formulas" page to be useful.

Problem 3 (12 points)

Consider the FIR filter $h[n] = -\delta[n] + 2\delta[n-1] - \delta[n-2]$.

(a) **(6 points):** Calculate the frequency response, $H_d(\omega)$, of this filter. Sketch $|H_d(\omega)|$ for $-\pi \leq \omega \leq \pi$. You may find the table of cosines and sines on the "useful formulas" page to be useful.

(b) **(3 points):** Is this a generalized linear phase filter? Why?

(c) **(3 points):** Is it a lowpass, highpass, bandpass, or bandstop filter? Why?

Problem 4 (12 points)

Use the windowing method to design a length-5 generalized linear phase FIR lowpass filter with a cutoff frequency of $\omega_c = \pi/4$.

(a) **(4 points):** An ideal lowpass filter with cutoff $\omega_c = \frac{\pi}{4}$ has the following Fourier transform:

$$D(\omega) = \begin{cases} 1 & |\omega| < \frac{\pi}{4} \\ 0 & |\omega| > \frac{\pi}{4} \end{cases}$$

Use the inverse Fourier transform formula to find $d[n]$, and sketch it as a function of n, for $-5 \leq n \leq 5$.

(b) **(4 points):** Choose any causal window of length $N = 5$ samples. Specify the name of the window you are using, and the mathematical formula used to calculate its coefficients. Sketch the window, $w[n]$, as a function of n, for $-5 \leq n \leq 5$.

(c) **(4 points):** Specify $h[n]$ in terms of $w[n]$ and $d[n]$ (that is, give an equation of the form "$h[n] = \ldots$" where "$\ldots$" is replaced by some function of $w[n]$ and $d[n]$). Sketch $h[n]$ as a function of n for $-5 \leq n \leq 5$.

Problem 5 (10 points)

You have recorded an electrocardiogram signal, $x[n]$, with a sampling frequency of $F_s = 1.2kHz$. Unfortunately, it has been corrupted by power line noise: it has a big sinusoidal component at 60Hz. Fortunately, you know how to eliminate power line noise using a notch filter. All you have to do is to pass the signal through a difference equation:

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2] - a_1 y[n-1] - a_2 y[n-2] \tag{1}$$

Use a pole amplitude of 0.98. What are b_1, b_2, a_1, and a_2?

(Note: leave your answer in the form of an explicit numerical expression. For example, if you discover that $a_1 = (0.3)^2 \sin(2\pi/400)$, then you should leave it in that form instead of trying to simplify.)

Problem 6 (12 points)

Consider the signal $x(t) = -2 + \sin(40\pi t)$

(a) **(2 points):** Determine and list all of the analog frequencies in the signal $x(t)$. Include negative frequencies.

(b) **(2 points):** What is the lowest possible sampling frequency that would avoid aliasing?

(c) **(2 points):** What is the corresponding Nyquist frequency for this sampling rate?

(d) **(2 points):** For a sampling frequency of $F_s = 100$Hz, find $x[n]$.

(e) **(2 points):** Determine and list all of the frequencies ω, $-\pi < \omega \leq \pi$, present in the discrete-time signal $x[n]$. Include negative frequencies.

(f) **(2 points):** If we take a length-20 DFT of 20 samples of this signal, for which values of k, $0 \leq k \leq 19$, will the DFT samples $X[k]$ be nonzero?

Problem 7 (12 points)

Assume that $x[n] = x_c(nT)$, where $F_s = 4000$ samples/second. For each of the following signals, find $x[n]$, $X_c(\Omega)$, and $X_d(\omega)$.

(a) **(6 points):** $x_c(t) = \text{sinc}(1000\pi t)$ (sinc, not sin!) HINT: Guess the form of $X_c(\Omega)$ and $X_d(\omega)$, then use the inverse CTFT and inverse DTFT formulas to find the amplitude of the nonzero part.

(b) **(6 points):** $x_c(t) = \cos(400\pi t)$. HINT: Guess the form of $X_c(\Omega)$ and $X_d(\omega)$, then use the inverse CTFT and inverse DTFT formulas to find the amplitude of the nonzero part.

Problem 8 (10 points)

Consider the following signal:

$$x[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]$$

(a) **(2 points):** Find and sketch $|X_d(\omega)|$, the magnitude DTFT of $x[n]$. You may find the table of cosines and sines on the "useful formulas" page to be useful.

(b) **(2 points):** Define $y_0(t)$ to be the piece-wise constant interpolation of $x[n]$ with a sampling rate of T seconds/sample. Sketch $y_0(t)$ for $0 \le t \le 4T$.

(c) **(2 points):** Define $y_1(t)$ to be the piece-wise linear interpolation of $x[n]$ with a sampling rate of T seconds/sample. Sketch $y_1(t)$ for $0 \leq t \leq 4T$.

(d) **(2 points):** Define $y_s(t)$ to be the sinc interpolation of $x[n]$ with a sampling rate of T seconds/sample. Sketch $y_s(t)$ for $0 \leq t \leq 4T$.

(e) **(2 points):** Define $y_s(t)$ to be the sinc interpolation of $x[n]$ with a sampling rate of T seconds/sample. Sketch $|Y_s(\Omega)|$, the CTFT of $y_s(t)$.

Problem 9 (6 points)

Determine whether the following system is linear and/or shift invariant:

$$y[n] = x[n]^2$$

(a) **(1 point):** Is it linear? YES, NO

(b) **(2 points):** Prove your answer to part (a):

(c) **(1 point):** Is it shift invariant? YES, NO

(d) **(2 points):** Prove your answer to part (c):

Problem 10 (6 points)

Determine whether the following LTI system is causal and/or BIBO stable:

$$y[n] = x[n+1] + y[n-1]$$

(a) **(1 point):** Is it causal? YES, NO

(b) **(2 points):** Prove your answer to part (a) (HINT: this proof can be given in words; you need not necessarily use any formulas)

(c) **(1 point):** Is it stable? YES, NO

(d) **(2 points):** Prove your answer to part (c)