ECE 401 Signal and Image Analysis
Homework 5

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Tuesday, 11/2/2021; Due: Monday, 11/8/2021
Reading: *DSP First* Chapter 8

Problem 5.1

Consider the signal $x[n] = \delta[n]+\delta[n-2]$. Plot the magnitude DTFT, $|X(\omega)|$, of this signal, for $0 \le \omega < 2\pi$. Draw circles on your plot to show the frequency samples $X[k]$ for a 4-point DFT.

Problem 5.2

In this problem, we will repeat Hamming's famous calculation, that resulted in the Hamming window. Consider a slightly modified, even-symmetric raised-cosine window,

$$w_C[n] = \left((1-a) + a\cos\left(\frac{2\pi n}{N}\right)\right) w_R[n]$$

where a is an arbitrary constant, whose value has not yet been determined, and $w_R[n]$ is

$$w_R[n] = \begin{cases} 1 & -M \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

and the total length of the window is $N = 2M+1$. Recall that the DTFT of an even-symmetric rectangular window is

$$W_R(\omega) = D_N(\omega) = \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

(a) Use the linearity and frequency-shift properties of the DTFT to find $W_C(\omega)$, the DTFT of $w_C[n]$.

(b) Sketch $W_C(\omega)$, for $0 \le \omega \le \frac{10\pi}{N}$. Draw circles at the frequencies that would be sampled by an N-point DFT. Find the values of $W_C[k]$ for all k in the range $0 \le k \le N-1$, as functions of a and N.

(c) Find $W_C\left(\frac{5\pi}{N}\right)$ in terms of a and N, and then find the value of a that zeros it out, $W_C\left(\frac{5\pi}{N}\right) = 0$.

Note: in order to find the value of $W_C\left(\frac{5\pi}{N}\right)$, you will want to take advantage of the fact that, for small enough values of k,

$$\frac{\sin(k\pi/2)}{\sin(k\pi/2N)} \approx \frac{\sin(k\pi/2)}{k\pi/2N} = \begin{cases} \pm\frac{2N}{k\pi} & k \text{ odd} \\ 0 & k \text{ even and nonzero} \end{cases}$$