ECE 401 Signal and Image Analysis
Homework 4

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Monday, 10/12/2020; Due: Monday, 10/19/2020
Reading: *DSP First* Sections 9.1-9.6, 10.1-10.3

Problem 4.1

Consider the difference equation:

$$y[n] = x[n] - \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$$

Find the frequencies, $\omega = \angle z_1$ and $\omega = \angle z_2$, of the two zeros.

Solution: The transfer function is

$$H(z) = 1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}$$

which has zeros at

$$z = \frac{1}{4} \pm j\frac{\sqrt{3}}{4}.$$

The frequencies of these two zeros are

$$\omega = \pm\frac{\pi}{3}\ \frac{\text{radians}}{\text{sample}}$$

Problem 4.2

A particular filter has the impulse response

$$h[n] = \begin{cases} 0.5 & n = 0 \\ 0.75 & n = 1 \\ 0.3 & n = 2 \\ 0.1 & n = 3 \\ 0 & \text{otherwise} \end{cases}$$

What is the transfer function, $H(z)$?

Solution:

$$H(z) = 0.5 + 0.75z^{-1} + 0.3z^{-2} + 0.1z^{-3}$$

Problem 4.3

A particular filter has the difference equation

$$y[n] = x[n] - 1.2e^{j3\pi n/5}x[n-1] + 0.8e^{j2\pi n/5}y[n-1]$$

Express the frequency response of this filter as

$$H(\omega) = \frac{e^{j\omega} - z_1}{e^{j\omega} - p_1}$$

for some zero z_1 and pole p_1.

Solution:

$$H(\omega) = \frac{e^{j\omega} - 1.2e^{j3\pi/5}}{e^{j\omega} - 0.8e^{j2\pi/5}}$$

Problem 4.4

Remember that

$$G(z) = \frac{1}{1 - 0.8z^{-1}} \leftrightarrow g[n] = (0.8)^n u[n]$$

Use the linearity and time-shift properties of the Z-transform to find $h[n]$, where

$$H(z) = \frac{1 - 0.3z^{-1}}{1 - 0.8z^{-1}} = \frac{1}{1 - 0.8z^{-1}} - 0.3z^{-1}\frac{1}{1 - 0.8z^{-1}}$$

Solution: The time-shift property is

$$z^{-n_0}G(z) \leftrightarrow g[n - n_0],$$

so

$$G(z) - 0.3z^{-1}G(z) \leftrightarrow g[n] - 0.3g[n-1],$$

therefore

$$h[n] = (0.8)^n u[n] - 0.3(0.8)^{n-1}u[n-1]$$