ECE 401 Signal and Image Analysis
Homework 2

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Tuesday, 9/1/2020; Due: Monday, 9/14/2020
Reading: *DSP First* pp. 12-34, 50-58, 61-71

Problem 2.1

In standard tuning, the middle A note on a piano (A4) has a frequency of 440Hz. Consider the note

$$x(t) = 14\cos\left(2\pi 440t + 0.88\pi\right)$$

Sketch one complete period of $x(t)$, from its first peak after $t = 0$ until its second peak after $t = 0$. Label the times of both peaks, and the value of $x(t)$ at both peaks.

Problem 2.2

Suppose you're given the signal

$$x(t) = \cos(2\pi 440t) + 3\sin(2\pi 440t)$$

Find the phasor representation of $x(t)$, and simplify it to polar form. You might want to take advantage of facts like $\sin(x) = \cos(x - \frac{\pi}{2})$, and $\sin(\frac{\pi}{2}) = 1$, and $\cos(\frac{\pi}{2}) = 0$.

Problem 2.3

Kwikwag's beat-tones example on Wikipedia adds two tones, at the frequencies 110Hz and 104Hz:

$$x(t) = \cos(2\pi 110t) + \cos(2\pi 104t)$$

Find a sequence of frequencies and phasors, $\{(f_{-2}, a_{-2}), \ldots, (f_2, a_2)\}$, such that

$$x(t) = \sum_{k=-2}^{2} a_k e^{j2\pi f_k t}$$

Problem 2.4

Suppose that a violin is playing the note A4 (440Hz), but our recording quality is bad, so we only get the first two harmonics:

$$x(t) = \sum_{k=-2}^{2} a_k e^{j2\pi k440t}$$

Suppose we measure the spectrum, and find it to be

$$\{(-880, 0.01), (-440, 1), (0, 0), (440, 1), (880, 0.01)\}$$

In order to improve the balance a little, we try differentiating the tone. Our differentiator also imposes a delay and a DC offset, though, so what we get is

$$y(t) = \frac{dx(t - 0.001)}{dt} + 1.5$$

Find the spectrum of $y(t)$.