ECE 398GG

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Homework 5 Solution

1. **Convert** the following instantaneous currents into phasors, using $cos\ (\omega t)$ as the reference. **State** your answers in polar form.

(i) $i(t) = 400\sqrt{2}\cos(\omega t - 30°)$

(ii) $i(t) = 5\, sin\,(\omega t + 15°)$

(iii) $i(t) = 4\cos(\omega t - 30°) + 5\sqrt{2}\sin(\omega t + 15°)$

Solution: We make use of the relationship that $sin\,(\mu) = cos\,(90° - \mu)$

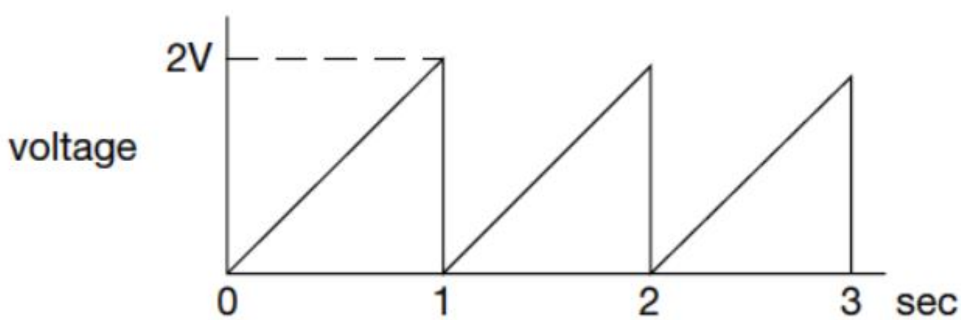
(i) $\left[400\sqrt{2}/\sqrt{2}\right]\angle -30° = 400\angle -30°$

(ii) $\left(5/\sqrt{2}\right)\angle -75°$

(iii)

$$\left(4/\sqrt{2}\right)\angle -30° + \left(5\sqrt{2}/\sqrt{2}\right)\angle -75° = \left(2\sqrt{2}\right)\angle -30° + 5\angle -75° = 7.3\angle -59°$$

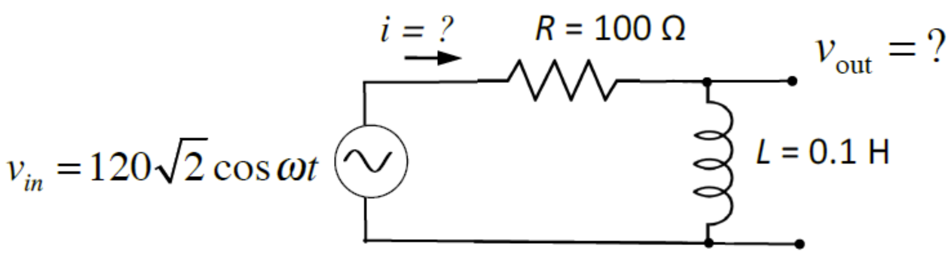
2. **Detemine** the r.*m.s.* value of voltage for the sawtooth waveform below.



Solution: We define $f(x) = 2x$ and compute the rms value for an arbitrary period

$$\sqrt{\int_0^1 [\, \frac{1}{1} * (2x)^2 \,]\, dx} = \frac{2}{\sqrt{3}}$$

3. For the following 120-*V*, 60-*Hz* circuit, perform the following:



(i) **Determine** the reactance and the impedance of the inductor?

(ii) **Express** the impedance of the combination of R and L in both polar coordinates and rectangular $Z = R + jX$ form.

(iii) **Determine** the current expressed as a phasor and as a function of time.

(iv) **Determine** the power factor of the circuit.

(v) **Calculate** the output voltage expressed as a phasor and as a function of time.

Solution:

(i)

$$X_L = \omega L = 2\pi \cdot 60 \cdot 0.1 = 37.7\Omega$$
$$\mathbf{Z}_L = j\omega L = j37.7 = 37.7\angle 90^o$$

(ii)

$$\mathbf{Z}_{Tot} = R + j\omega L = 100 + j37.7$$
$$= \sqrt{100^2 + 37.7^2}\angle \tan^{-1}\left(\frac{37.7}{100}\right) = 106.9\angle 20.66^o$$

(iii) the angle –20.66 is written incorrectly in the lines after the first equation

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \frac{120\angle 0°}{106.9\angle 20.66°} = 1.12\angle\text{-}20.66°$$
$$i = 1.12\sqrt{2}\cos\left(\omega t\text{-}20.22^0\right) = 1.12\sqrt{2}\cos\left(377t\text{-}20.22^0\right)$$

(*iv*) *p.f.* = *cos* (–20.66) = 0.94

(v) The two evaluations

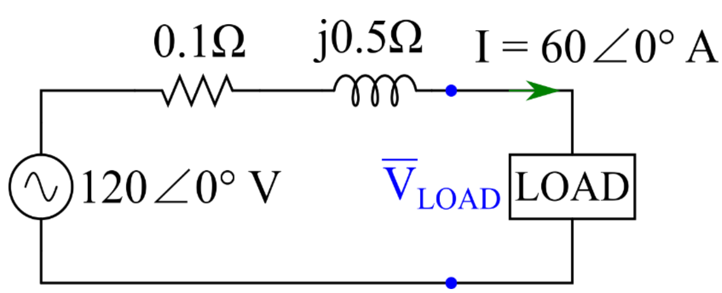
$$\boldsymbol{V}_{out} = \boldsymbol{Z}_L \boldsymbol{I} = j37.7x1.12\angle -20.66 = 42.2\angle 69.3$$

OR...using the voltage divider approach:

$$\boldsymbol{V}_{out} = \boldsymbol{V}_{in} \cdot \frac{\boldsymbol{Z}_L}{\boldsymbol{Z}_R + \boldsymbol{Z}_L} = 120\angle 0^o \cdot \frac{37.7\angle 90^o}{106.9\angle 20.66^o} = 42.3\angle 69.3^o$$

$$v_{out} = 42.3\sqrt{2}\cos\left(\omega t + 69.3^o\right)$$

4. For the circuit shown below, perform the following:



(i) **Compute** the voltage across the load terminals

(ii) **Determine** the real power delivered to the load in terms of the voltage and current phasors at the load.

(iii) **Calculate** the real power delivered to the load via the power flow equation derived in class, under the assumption of a lossless transmission line.

(iv) **Comment** on the differences, if any, between the results in (ii) and (iii).

Solution:

(i) V_{load} = 120 Ð 0 – 60 Ð 0 x (0.1 + *j*0.5) = 117 Ð – 14.7°

(ii) $P = V$ x I x *cos* ($\theta_V - \theta_I$) = 117 x 60 x *cos* (–14.7° – 0) = 6,790 *W*

(iii) P = [(V_1 x V_2)/ *x*]*sin*($\theta_1 - \theta_2$) = 7,150 *W*

(iv) Given that the power flow relation assumes a lossless line and this is not the case since there is a 0.1 *ohm* resistance, the answers are relatively close and so the order of magnitude of the difference is acceptable in this case.